

Phase Transitions and Critical Phenomena



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Exercise Sheet 5

HS 14
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Problem 1 Scaling laws

The renormalization group method can be used to derive the scaling of the free energy of a magnetic system

$$F(t, h) = |t|^{d/y_t} f^{\pm} \left(\frac{h}{|t|^{y_h/y_t}} \right). \quad (1)$$

Use equation (1) and the definition of critical exponents $\alpha, \beta, \gamma, \delta$ to derive

- the Rushbrooke law

$$\alpha + 2\beta + \gamma = 2, \quad (2)$$

- the Widom law

$$\beta\delta = \beta + \gamma. \quad (3)$$

Problem 2 Ginzburg criterion

A system close to a phase transition has two relevant length scales: The correlation length ξ and scale r_T on which thermal fluctuations of the order parameter are comparable to the order parameter.

Show that in the Landau theory $\xi \simeq r_T$ corresponds to the Ginzburg criterion, e.g.

$$t_G \simeq \left(\frac{1}{\xi_0^d \Delta c_V} \right)^{\frac{2}{4-d}}. \quad (4)$$