Exercise 1. *Helicity method*

(a) Show

$$\langle 1-|\gamma^\mu|2-\rangle\langle 3+|\gamma_\mu|4+\rangle = 2\,\langle 14\rangle[32]$$

Hint: multiply the l.h.s. by $1 = \langle 23\rangle/\langle 23\rangle$

(b) Show the equivalence

$$\langle 1-|\gamma^\mu|2-\rangle\,\gamma_\mu = 2\,(|1+\rangle\langle 2+| + |2-\rangle\langle 1-|)$$

by contraction with arbitrary spinors.

(b) Prove the Schouten identity

$$\langle 12\rangle\langle 34\rangle = \langle 14\rangle\langle 32\rangle + \langle 13\rangle\langle 24\rangle$$

Hint: multiply the l.h.s. by $1 = [23]/[23]$.

Exercise 2. $0 \to e^+(p_1)\, e^-(p_2)\, \gamma(p_3)\, \gamma(p_4)$

The amplitudes for the process $0 \to e^+(p_1, h_1)\, e^-(p_2, h_2)\, \gamma(p_3, h_3)\, \gamma(p_4, h_4)$ (all particles outgoing) are denoted by $M(h_1, h_2, h_3, h_4)$ with $h_i \in \{+, -\}$.

(a) Show that due to helicity conservation along the fermion lines we have $M(+, +, h_3, h_4) = M(-, -, h_3, h_4) = 0$.

(b) Compute $M(+, -, +, +)$ for arbitrary reference momenta q_3 and q_4 and show that this amplitude vanishes as well.

(c) Compute $M(+, -, +, -)$ and $M(+, -, -, +)$ making suitable choices for the reference momenta q_3 and q_4. Use these results to compute the spin summed/averaged matrix element squared for Compton scattering.

(d)* Compute $M(+, -, +, -)$ for arbitrary reference momenta q_3 and q_4 and show that the result is independent of q_3 and q_4.

Exercise 3. *UV singularities in* ϕ^n

Given the Lagrangian

$$\mathcal{L} = \frac{1}{2}\,\partial^\mu\phi\,\partial_\mu\phi - \frac{m_0^2}{2}\,\phi^2 - \frac{\lambda_0}{n!}\,\phi^n$$

determine the superficial degree of divergence of a L-loop diagram with V vertices, E_b external lines and I_b internal lines. For what value of n is the theory super-renormalizable, renormalizable, non renormalizable?