

# Critical exponents

singular behavior at  $T=T_{c\pm}$

control parameter  $\tau = 1 - \frac{T}{T_c}$

$$\tau > 0, \tau < 0$$

heat capacity

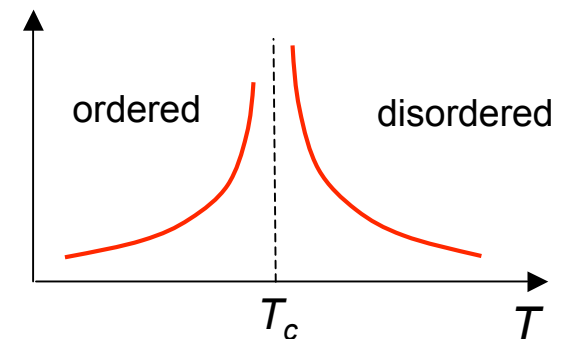
$$C(T) \propto |\tau|^{-\alpha}$$

susceptibility

$$\chi(T) \propto |\tau|^{-\gamma}$$

correlation length

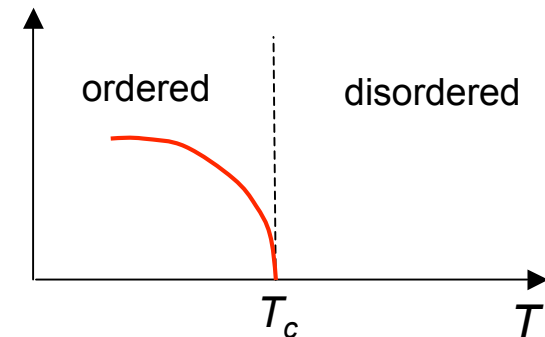
$$\xi(T) \propto |\tau|^{-\nu}$$



$$\tau > 0 \quad (T < T_c)$$

order parameter

$$m(T) \propto |\tau|^\beta$$



# Critical exponents

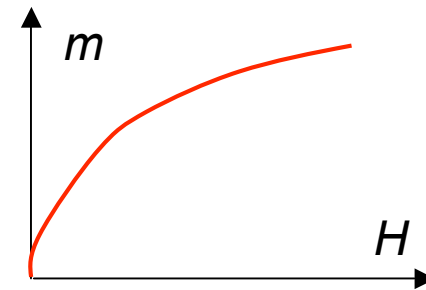
singular behavior at  $T=T_{c\pm}$

control parameter  $\tau = 1 - \frac{T}{T_c}$

$$\tau = 0 \quad (T = T_c)$$

order parameter  $m \propto H^{1/\delta}$

correlation function  $\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2+\eta}}$



## scaling laws

Rushbrooke scaling:  $\alpha + 2\beta + \gamma = 2$

Widom scaling:  $\gamma = \beta(\delta - 1)$

Fisher scaling:  $\gamma = (2 - \eta)\nu$

Josephson scaling:  $\nu d = 2 - \alpha$

# Critical exponents

Fisher scaling:  $\gamma = (2 - \eta)\nu$

general correlation function  $\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2+\eta}} g(r/\xi) \quad \xi(T) \propto |\tau|^{-\nu}$

susceptibility  $\chi(T) \propto |\tau|^{-\gamma}$

$$\begin{aligned} \chi &\propto \int d^d r \Gamma_{\vec{r}} \propto \int d^d r \frac{1}{r^{d-2+\eta}} g(r/\xi) \\ &\propto \xi^{2-\eta} \int d^d y \frac{1}{y^{d-2+\eta}} g(y) \propto |\tau|^{-\nu(2-\eta)} \end{aligned}$$

# Critical exponents

Mean field exponents:

$$-A'\tau m + Bm^3 - H - \kappa \vec{\nabla}^2 m = 0$$

$$\tau < 0 \quad (T > T_c) \quad \xi^2 = -\frac{\kappa}{A'\tau} \propto |\tau|^{-2\nu}$$

$$\chi = -\frac{1}{A'\tau} \propto |\tau|^{-\gamma}$$

$$\tau > 0 \quad (T < T_c) \quad m^2 = \frac{A'\tau}{B} \propto |\tau|^{2\beta}$$

$$\tau = 0 \quad (T = T_c) \quad Bm^3 = H \propto H^{3/\delta}$$

$$\Gamma_{\vec{r}} \propto \frac{1}{r^{d-2}} \propto \frac{1}{r^{d-2+\eta}}$$

$$C \propto \Theta(\tau) \propto |\tau|^{-\alpha}$$

$$\nu = \frac{1}{2}$$

$$\gamma = 1$$

$$\beta = \frac{1}{2}$$

$$\delta = 3$$

$$\eta = 0$$

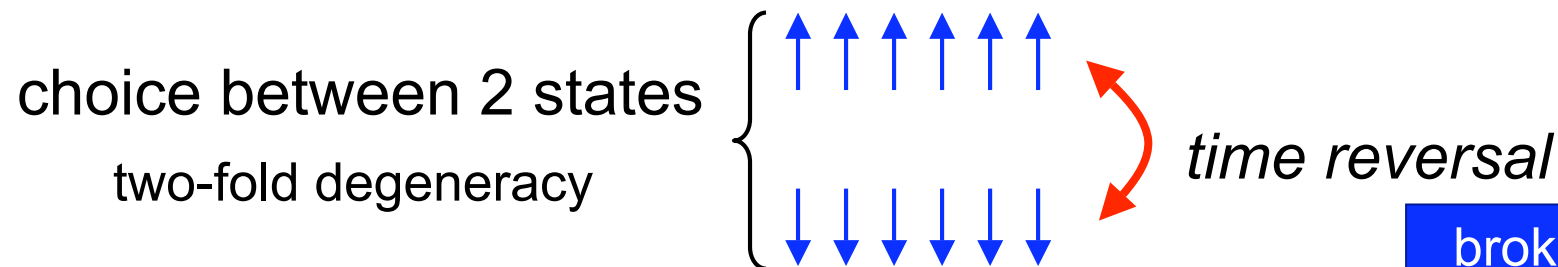
$$\alpha = 0$$

# Spontaneous symmetry breaking - long range order

Ginzburg-Landau theory (Ising model of ferromagnet)

order parameter  $m \propto \begin{cases} 0 & T > T_c \\ \pm |\tau|^{1/2} & T < T_c \end{cases}$

high symmetry  $\mathcal{G}$   
low symmetry  $\mathcal{G}'$



broken  
symmetry

free energy functional

$$F[m; H, T] = \int d^d r \left\{ \frac{A}{2} m(\vec{r})^2 + \frac{B}{4} m(\vec{r})^4 - H(\vec{r}) m(\vec{r}) + \frac{\kappa}{2} [\vec{\nabla} m(\vec{r})]^2 \right\}$$

scalar (invariant) under symmetry operations in  $\mathcal{G} = \mathcal{G} \times \mathcal{K}$

space group      time reversal

# Spontaneous symmetry breaking - long range order

correlation function

$$\Gamma_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \xrightarrow{\vec{r} = \vec{r}_j - \vec{r}_i} \Gamma_{\vec{r}} \propto \frac{e^{-r/\xi}}{r^b} \quad \text{for } \begin{matrix} T < T_c \\ T > T_c \end{matrix}$$

$$\lim_{r \rightarrow \infty} \Gamma_{\vec{r}} = 0 \quad \rightarrow \quad \lim_{r \rightarrow \infty} \langle s_i s_j \rangle = \langle s_i \rangle \langle s_j \rangle$$

$$T > T_c$$

$$\langle s_i \rangle = 0$$

$$\lim_{r \rightarrow \infty} \langle s_i s_j \rangle = 0$$

$$T < T_c$$

$$\langle s_i \rangle = \pm m$$

$$\lim_{r \rightarrow \infty} \langle s_i s_j \rangle = m^2 > 0$$

long range order

correlation over arbitrary distance