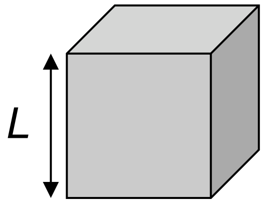
# Gas of free Fermions

Plane waves $\psi_{\vec{k}}(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$ with
$$\begin{cases} \text{momentum} & \vec{p} = \hbar\vec{k} \\ \text{energy} & \epsilon_{\vec{p}} = \frac{\vec{p}^2}{2m} = \frac{\hbar^2\vec{k}^2}{2m} \end{cases}$$

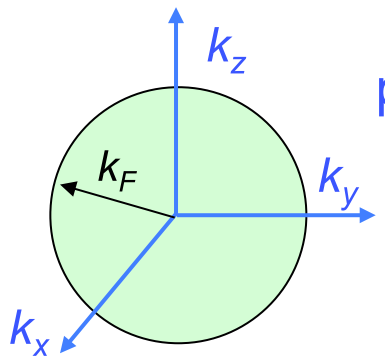



$$V = L^3$$

periodic boundary conditions

$$\psi_{\vec{k}}(x,y,z) = \psi_{\vec{k}}(x+L,y,z) = \psi_{\vec{k}}(x,y+L,z) = \psi_{\vec{k}}(x,y,z+L)$$

$$\Rightarrow \quad \vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z) = \frac{2\pi}{L}\vec{n} \quad \text{with} \quad n_{x,y,z} = 0, \pm 1, \pm 2, \ldots$$




particle number / density

$$N = \sum_{\vec{n}}^{occ.} 2 = 2\frac{L^3}{(2\pi)^3}\int_{|\vec{k}|\le k_F} d^3k = \frac{2V}{(2\pi)^3}\frac{4\pi}{3}k_F^3 \qquad n = \frac{N}{V}$$

$$\sum_{\vec{n}} \rightarrow \frac{V}{(2\pi)^3}\int d^3k$$

$\Rightarrow$ *Fermi wavevector* $\quad k_F = \{3\pi^2 n\}^{1/3}$

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$$\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z) = \frac{2\pi}{L}\vec{n} \qquad k_F = \{3\pi^2 n\}^{1/3} = \{3\pi^2 N/V\}^{1/3}$$

distribution function

$$f_0(\epsilon) = 2\frac{1}{h^3}\frac{1}{e^{\beta(\epsilon-\mu)}+1}$$

"renormalization"

spin

$$n = \int d^3p\, f_0(\epsilon_{\vec{p}}) = 4\pi \int dp\, p^2 f_0(\epsilon_{\vec{p}}) = 4\pi\sqrt{2m^3} \int d\epsilon \epsilon^{1/2}\, f_0(\epsilon)$$

$$n = \int d\epsilon\, g(\epsilon) f_0(\epsilon) \quad \text{with} \quad g(\epsilon) = \frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}$$

*density of states*