Linear response - harmonic oscillator

$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{x}^2$$

$$\mathcal{H}|n\rangle = \epsilon_n|n\rangle \qquad \epsilon_n = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

$$\left\{ \begin{array}{c} \hat{a}|n\rangle = \sqrt{n}|n-1\rangle \\ \\ \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \end{array} \right\}$$

$$\hat{x} = \frac{x_0}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger) \qquad \hat{p} = \frac{-i\hbar}{x_0\sqrt{2}}(\hat{a} - \hat{a}^\dagger)$$

$$x_0 = \sqrt{\hbar/m\omega_0}$$

perturbation:

$$\mathcal{H}' = -h(t)\hat{x}$$

$$\chi(t-t') = -\frac{i}{\hbar}\Theta(t-t')\langle[\hat{x}_H(t), \hat{x}_H(t')]\rangle_{\mathcal{H}}$$

$$\chi(\omega) = \sum_{n,n'} \frac{e^{-\beta\epsilon_n}}{Z}|\langle n|\hat{x}|n'\rangle|^2 \left\{ \frac{1}{\hbar\omega - \epsilon_{n'} + \epsilon_n + i\hbar\eta} - \frac{1}{\hbar\omega - \epsilon_n + \epsilon_{n'} + i\hbar\eta} \right\}$$

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$$\langle n|\hat{x}|n'\rangle = \frac{x_0}{\sqrt{2}}\langle n|(\hat{a}+\hat{a}^\dagger)|n'\rangle = \frac{x_0}{\sqrt{2}}\left\{\delta_{n',n+1}\sqrt{n+1}+\delta_{n',n-1}\sqrt{n}\right\}$$

$$\chi(\omega) = \frac{x_0^2}{2\hbar}\left\{\frac{1}{\omega-\omega_0+i\eta}-\frac{1}{\omega+\omega_0+i\eta}\right\}$$

dynamical structure factor $S(\omega) = \sum_{n,n'}\frac{e^{-\beta\epsilon_n}}{Z}|\langle n|\hat{x}|n'\rangle|^2\delta(\hbar\omega-\epsilon_{n'}+\epsilon_n)$

$$S(\omega) = \frac{x_0^2}{2}\{\langle n+1\rangle\delta(\hbar\omega-\hbar\omega_0)+\langle n\rangle\delta(\hbar\omega+\hbar\omega_0)\}$$

$$S(\omega) = \frac{x_0^2}{2}\frac{1}{e^{\beta\hbar\omega_0}-1}\{e^{\beta\hbar\omega_0}\delta(\hbar\omega-\hbar\omega_0)+\delta(\hbar\omega+\hbar\omega_0)\}$$

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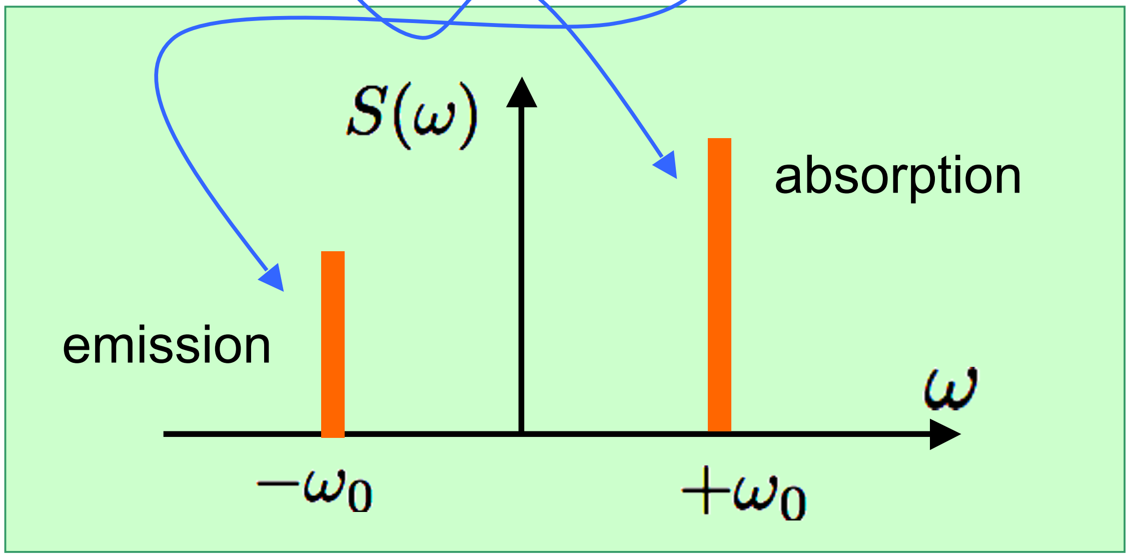
$$S(\omega) = \frac{x_0^2}{2}\frac{1}{e^{\beta\hbar\omega_0}-1}\{e^{\beta\hbar\omega_0}\delta(\hbar\omega-\hbar\omega_0)+\delta(\hbar\omega+\hbar\omega_0)\}$$

absorption
emission
rates



$$\chi''(\omega) = -\pi[S(\omega)-S(-\omega)] = -\frac{\pi x_0^2}{2}[\delta(\hbar\omega-\hbar\omega_0)-\delta(\hbar\omega+\hbar\omega_0)]$$

dissipative power

$$h(t) = \tilde{h}(\omega)\cos(\omega t)$$

$$\langle \dot{E}\rangle(\omega) = \frac{\omega}{2}\chi''(\omega)|\tilde{h}(\omega)|^2$$

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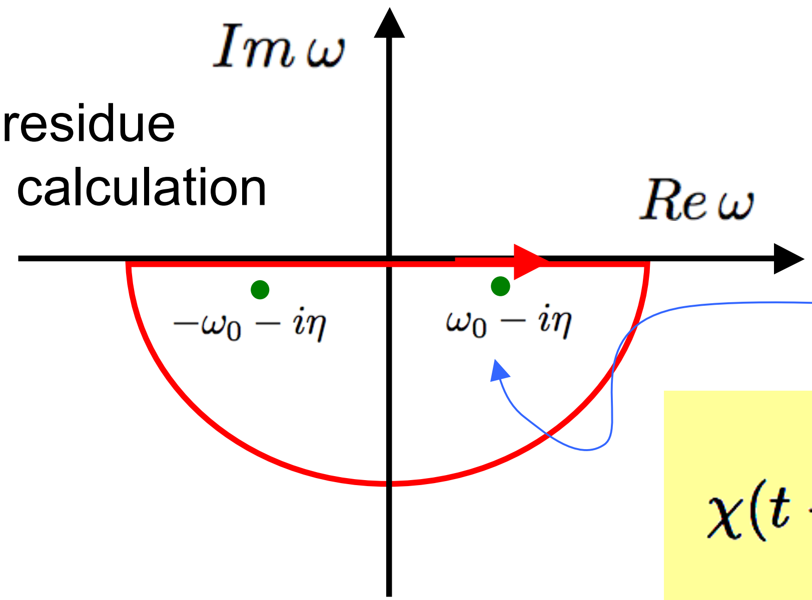
time dependence: $$\langle \hat{x} \rangle (t) = \int dt' \, \chi(t-t') h(t')$$

$$\chi(t) = \int \frac{d\omega}{2\pi} \chi(\omega) e^{-i\omega t} = \frac{x_0^2}{4\pi\hbar} \int d\omega \left\{ \frac{e^{-i\omega t}}{\omega - \omega_0 + i\eta} - \frac{e^{-i\omega t}}{\omega + \omega_0 + i\eta} \right\}$$

residue calculation



causality

$$\chi(t-t') = -\frac{x_0^2}{\hbar} \sin[\omega_0 (t-t')] \Theta(t-t')$$