

Electromagnetic wave - harmonic oscillator

$$\begin{aligned}\vec{E} &= -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} & \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \vec{A} &= 0 & \vec{\nabla} \cdot \vec{A} &= 0 \\ \vec{B} &= \vec{\nabla} \times \vec{A}\end{aligned}$$

plane waves in cavity ($L \times L \times L$):

$$\begin{aligned}\vec{A}(\vec{r}, t) &= \frac{1}{\sqrt{V}} \sum_{\vec{k}, \lambda} \left\{ A_{\vec{k}\lambda} \vec{e}_{\vec{k}\lambda} e^{i\vec{k} \cdot \vec{r} - i\omega t} + A_{\vec{k}\lambda}^* \vec{e}_{\vec{k}\lambda}^* e^{-i\vec{k} \cdot \vec{r} + i\omega t} \right\} \\ \omega &= \omega_{\vec{k}} = c|\vec{k}| & \vec{e}_{\vec{k}\lambda} \cdot \vec{k} &= 0\end{aligned}$$

Periodic boundary conditions:

$$\vec{k} = \frac{2\pi}{L} (n_x, n_y, n_z) \quad n_i = 0, \pm 1, \pm 2, \dots$$

Electromagnetic wave - Canonical quantization

$$Q_{\vec{k}\lambda} = \frac{1}{\sqrt{4\pi c}} (A_{\vec{k}\lambda} + A_{\vec{k}\lambda}^*) \quad P_{\vec{k}\lambda} = \frac{i\omega_{\vec{k}}}{\sqrt{4\pi c}} (A_{\vec{k}\lambda} - A_{\vec{k}\lambda}^*)$$

$$\text{Hamiltonian: } \mathcal{H} = \int d^3r \frac{\vec{E}^2 + \vec{B}^2}{8\pi} = \sum_{\vec{k}, \lambda} \frac{\omega_{\vec{k}}}{2\pi c} |A_{\vec{k}\lambda}|^2 = \frac{1}{2} \sum_{\vec{k}, \lambda} (P_{\vec{k}\lambda}^2 + \omega_{\vec{k}}^2 Q_{\vec{k}\lambda}^2)$$

canonical quantization

$$[Q_{\vec{k}, \lambda}, P_{\vec{k}', \lambda'}] = i\hbar \delta_{\vec{k}\vec{k}'} \delta_{\lambda\lambda'}$$

raising / lowering operators

$$A_{\vec{k}\lambda}^* \rightarrow a_{\vec{k}\lambda}^\dagger$$

$$A_{\vec{k}\lambda} \rightarrow a_{\vec{k}\lambda}$$

$$[a_{\vec{k}\lambda}, a_{\vec{k}', \lambda'}^\dagger] = \delta_{\vec{k}\vec{k}'} \delta_{\lambda\lambda'}$$

$$\mathcal{H} = \sum_{\vec{k}, \lambda} \hbar \omega_{\vec{k}} \left(a_{\vec{k}\lambda}^\dagger a_{\vec{k}\lambda} + \frac{1}{2} \right) = \sum_{\vec{k}, \lambda} \hbar \omega_{\vec{k}} \left(n_{\vec{k}\lambda} + \frac{1}{2} \right)$$

create bosonic particles in mode

(\vec{k}, λ) with $\omega_{\vec{k}}$ **photon**