# spin wave in ferromagnet - magnon

## Heisenberg ferromagnet

$$\mathcal{H} = -J\sum_{\langle i,j\rangle}\hat{\vec{S}}_i\cdot\hat{\vec{S}}_j = -J\sum_{\langle i,j\rangle}\left[\hat{S}_i^z\hat{S}_j^z + \frac{1}{2}\left\{\hat{S}_i^+\hat{S}_j^- + \hat{S}_i^-\hat{S}_j^+\right\}\right]$$

ground state: $\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$ $\quad |\psi^{(0)}\rangle = |\downarrow\downarrow\cdots\downarrow\downarrow\rangle$

$$\langle\hat{S}_i^z\rangle = -\frac{\hbar}{2}$$

$$\mathcal{H}|\psi^{(0)}\rangle = -\frac{J\hbar^2 N}{4}|\psi^{(0)}\rangle = E_0|\psi^{(0)}\rangle$$

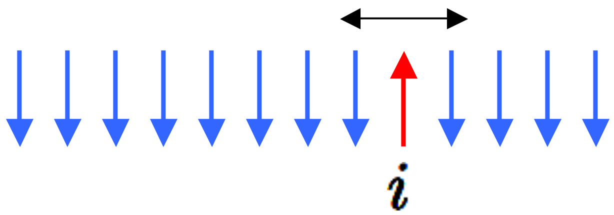
excited state: $\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\underset{i}{\uparrow}\downarrow\downarrow\downarrow\downarrow$ $\quad |\psi_i^{(1)}\rangle$

$$\mathcal{H}_1 = -J\sum_{\langle i,j\rangle}\hat{S}_i^z\hat{S}_j^z \rightarrow \mathcal{H}_1|\psi_i^{(1)}\rangle = E_1|\psi_i^{(1)}\rangle$$

$$E_1 = E_0 + \frac{Jz\hbar^2}{2}$$

# spin wave in ferromagnet - magnon

excited state: $\mathcal{H}_1 = -J \sum_{\langle i,j \rangle} \hat{S}_i^z \hat{S}_j^z$ ➡ $\mathcal{H}_1 |\psi_i^{(1)}\rangle = E_1 |\psi_i^{(1)}\rangle$




$|\psi_i^{(1)}\rangle$

$$E_1 = E_0 + \frac{Jz\hbar^2}{2}$$

$$\left.\begin{array}{l} \mathcal{H}_2 = -\frac{J}{2} \sum_{\langle i,j \rangle} \left\{ \hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ \right\} \\ \hat{S}_i^+ | \downarrow \rangle = \hbar | \uparrow \rangle \quad \hat{S}_i^- | \uparrow \rangle = \hbar | \downarrow \rangle \end{array}\right\} \Rightarrow$$

"spin hopping"

$$\mathcal{H}_2 |\psi_i^{(1)}\rangle = -\frac{J\hbar^2}{2} \sum_{j \in \Lambda_i} |\psi_j^{(1)}\rangle$$

nearest neighbors of *i*

Fourier transformation $|\phi_{\vec{q}}^{(1)}\rangle = \frac{1}{V} \sum_i |\psi_i^{(1)}\rangle e^{-i\vec{q}\cdot\vec{r}_i}$ "moving particle"

$$\Rightarrow \mathcal{H}_2 |\phi_{\vec{q}}^{(1)}\rangle = -\frac{J\hbar^2}{2} \sum_{j \in \Lambda_i} e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} |\phi_{\vec{q}}^{(1)}\rangle = \frac{J\hbar^2}{2} \gamma_{\vec{q}} |\phi_{\vec{q}}^{(1)}\rangle$$

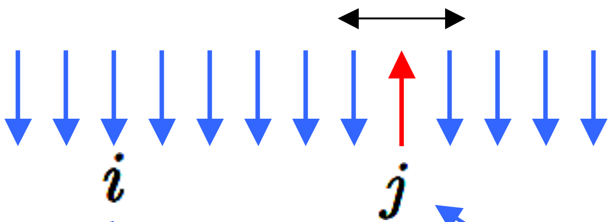
# spin wave in ferromagnet - magnon

$$\left.\begin{aligned}\mathcal{H}_1|\phi^{(1)}_{\vec{q}}\rangle &= \left(E_0+\frac{Jz\hbar^2}{2}\right)|\phi^{(1)}_{\vec{q}}\rangle \\ \mathcal{H}_2|\phi^{(1)}_{\vec{q}}\rangle &= \frac{J\hbar^2}{2}\gamma_{\vec{q}}|\phi^{(1)}_{\vec{q}}\rangle\end{aligned}\right\} \quad \mathcal{H}|\phi^{(1)}_{\vec{q}}\rangle = \left(E_0+\frac{J\hbar^2}{2}(\gamma_0-\gamma_{\vec{q}})\right)|\phi^{(1)}_{\vec{q}}\rangle$$

$$\gamma_{\vec{q}} = 2\sum_{\alpha=x,y,z}\cos q_\alpha a$$

$$|\phi^{(1)}_{\vec{q}}\rangle = \frac{1}{V}\sum_i |\psi^{(1)}_i\rangle e^{-i\vec{q}\cdot\vec{r}_i}$$

$$\Delta E_{\vec{q}} = J\hbar^2\sum_{\alpha=x,y,z}(1-\cos q_\alpha a)$$




flip down

measurement

flip up

probe

$$\chi_{ij}(t) = \frac{i}{\hbar}\Theta(t)\langle[\hat{S}^-_i(t),\hat{S}^+_j(0)]\rangle$$