

Exercise 13.1 Critical temperature in the Stoner model

We consider three types of dispersion relations:

- $\epsilon_{\mathbf{k}} = \epsilon_0 \pm \frac{\hbar^2 \mathbf{k}^2}{2m}$ (3D) and
- $\epsilon_k = \epsilon_0 + \alpha k$ (1D).

Plot the critical temperature of the Stoner model for fixed interaction strength U depending on the chemical potential μ .

Exercise 13.2 Stoner instability

In the lecture, it was shown that a system described by the mean-field Hamiltonian

$$\mathcal{H}_{\text{MF}} = \frac{1}{\Omega} \sum_{\mathbf{k}, s} (\epsilon_{\mathbf{k}} + U n_{-s}) c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s} - U n_\uparrow n_\downarrow \quad (1)$$

shows an instability towards a magnetically ordered state at $N(\epsilon_F)U_C = 1$ (note that here, $N(\epsilon)$ is the density of states per spin).

Show for the case of a parabolic dispersion and $T = 0$ that there are actually three distinct states:

- a paramagnetic state: $N(\epsilon_F)U < 1$,
- an imperfect ferromagnetic state: $3/2^{4/3} > N(\epsilon_F)U > 1$ and
- a perfect ferromagnetic state: $N(\epsilon_F)U > 3/2^{4/3}$.

Hint: Introduce a variable for the magnitude of the polarization

$$\frac{N_\uparrow}{N_e} = \frac{1}{2}(1+x) \quad \frac{N_\downarrow}{N_e} = \frac{1}{2}(1-x) \quad (2)$$

where $N_{\uparrow(\downarrow)}$ is the total number of up-spins (down-spins) and N_e is the total number of electrons. Minimize the total energy of the system with respect to x .

Plot the polarization of the system x as a function of $N(\epsilon_F)U$.

Exercise 13.3 Particle-Hole Excitations in Itinerant Ferromagnets

In section 7.3 of the lecture notes the low-energy spectrum of (magnons) spin-waves in itinerant ferromagnets was derived. It is crucial for the existence of well-defined magnons that elementary particle-hole excitations are gapped. Try to explain (without detailed calculations) why there is such a gap and why it is important for the observability of magnons!

Office hour:

Monday, May 30th, 2011 - 9:00 to 11:00 am

HIT K 12.1

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