Thermodynamik
Serie 10

HS 07
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Aufgabe 10.1 Raising of the boiling point and lowering of the melting point

Consider the balance between a diluted solution and the steam of the solvent - we assume the solute not to be volatile, so that it does not go into the gaseous phase. Here the interface between the two phases acts as a semipermeable wall.

(a) Derive Raoult's law for the raising of the boiling point and for the lowering of the melting point (vgl. Skript S.94).

(b) Draw the corresponding $p - T$ diagram.

(c) How much salt would one need in order to cook an egg on the Mount Everest? (Albumen coagulates at $60°C$).

(d) When a thermodynamic system is perturbed, it tries to counter-act the imposed change (*Le Chatelier*'s principle). Discuss the form of the $p - T$ diagram when one disturbs the system by increasing the salt concentration.

Aufgabe 10.2 Mixture diagrams

(a) The two isotopes ^{3}He and ^{4}He can be mixed in any ratio only at a temperature above $0.8K$. Draw a qualitative mixture diagram for a mixture of ^{3}He and ^{4}He at temperatures below $1K$.

(b) Consider the equilibrium condition in the vicinity of the melting points and the behavior for $T \to 0$ for a copper-silver mixture.

Aufgabe 10.3 Landau theory of second-order phase transitions

In most cases, a phase transition is accompanied by a lowering ("breaking") of the symmetry of the system. The high temperature (disordered) phase has a higher symmetry than the (ordered) phase at lower temperature. In Landau theory, this aspect is taken care of by introducing an order parameter which increases from zero as the temperature drops below T_c. As a simple example, we consider in this exercise a (Heisenberg) ferromagnet where the order parameter is the magnetization density $\vec{m}$ which becomes finite below the critical temperature.
In Landau theory, we only consider the part of the free energy which is associated with the phase transition and, since the order parameter is small around T_c, expand it in powers of $\vec{m}$. As we want to describe a real system, we require the free energy to be analytic and incorporate the underlying symmetry. In the case of a ferromagnet, the symmetry is O(3) and we find

$$f(m,T) = a(T)m^2 + \frac{b(T)}{2}m^4 \tag{1}$$

with $m = |\vec{m}|$.

(a) As we know (from the case of paramagnets), we can create a finite magnetization above T_c by applying a (conjugate) magnetic field $\vec{h}$. Show that for $h = |\vec{h}| = 0$ we can find the magnetization by minimizing the free energy $f(m, T)$ with respect to m.
Hint: How can one change to the corresponding new potential?

(b) We expect m to be 0 for $T > T_c$ and finite for $T < T_c$. What conditions should we stipulate for $a(T)$ and $b(T)$ such that this behavior is fulfilled?

(c) We now set $a(T) = a_0(T - T_c)$, $a_0 > 0$ and $b(T) = b = \text{const} > 0$.
In the vicinity of a phase transition, one writes the scaling relations

- $m(T) \propto (T_c - T)^{\beta}$ ($h = 0$ and $T < T_c$),
- $c_V(T) \propto |T - T_c|^{-\alpha}$ ($h = 0$),
- $\chi(T) = \partial_h m|_{h=0} \propto |T - T_c|^{-\gamma}$ ($h = 0$) and
- $m(h) \propto h^{1/\delta}$ ($T = T_c$).

Calculate with our model the above quantities and thus the critical exponents α, β, γ and δ.