Simple picture of de Haas - van Alphen effect

The area of the orbit in K space is given by the Onsager equation

$$A_n = (n+\gamma)\,\frac{B}{\Phi_0}$$

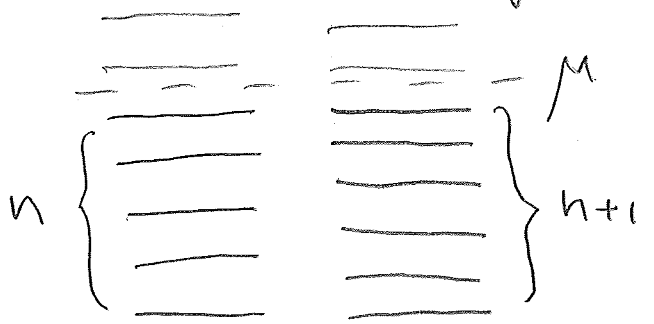
How much should we change the field such that $n+1$-st orbit has the same area as n-th?

$$A_n(B) = A_{n+1}(B-\delta B) \Rightarrow$$

$$(n+\gamma)\,B = (n+1+\gamma)(B-\delta B) \Rightarrow$$

$$\delta B = \frac{B}{n+\gamma+1} \Rightarrow \underline{-\delta\frac{1}{B} = \frac{\delta B}{B^2} = \frac{1}{B(n+\gamma+1)} = \frac{1}{\Phi_0 A_{n+1}}}$$

Consider filling of the Landau levels



Decreasing fields levels go down and after changing it by δB calculated above next level will go below the chemical potential. Then the distribution of levels would be very similar to the previous one, just we will have $n+1$ levels occupied instead of n. But since $n \gg 1$ then the difference would be small and we will get periodic changes of properties

To calculate magnetization consider a simplified model of a 2d metal in perpendicular field at $T=0$

Then we have discrete levels $\varepsilon_n = \hbar\omega_c (n+\frac{1}{2})$.

Degeneracy of every level is $\frac{BS}{\Phi_0} = \frac{\Phi}{\Phi_0}$.

If the number of electrons $N_e < \frac{\Phi}{\Phi_0}$ then all of them occupy the lowest Landau level with energy $\frac{\hbar\omega_c}{2} = \mu_B B$. The total energy would be

$E = N_e \mu_B B$ and the magnetization density

$$M = -\frac{1}{S}\frac{\partial E}{\partial B} = -n_e \mu_B, \quad n_e = \frac{N_e}{S} \text{ - electron density}$$

Decreasing the field such that

$$\frac{2\Phi}{\Phi_0} > N_e > \frac{\Phi}{\Phi_0} \iff \frac{n_e \Phi_0}{2} < B < n_e \Phi_0$$

will produce complete filling of the lowest Landau level and partial of the next with $n=1$

$$\frac{E}{S} = \mu_B B \frac{B}{\Phi_0} + 3\mu_B B\left(n_e - \frac{B}{\Phi_0}\right) = 3\mu_B n_e B - 2\mu_B \frac{B^2}{\Phi_0}$$

and $M = -3\mu_B n_e + 4\mu_B \frac{B}{\Phi_0}$

At $B = n_e \Phi_0$ magnetization jumps from $-\mu_B n_e$ to $+\mu_B n_e$

Then it will linearly decrease back to $-\mu_B n_e$ at $B = \frac{n_e \Phi_0}{2}$. Consider then the field range

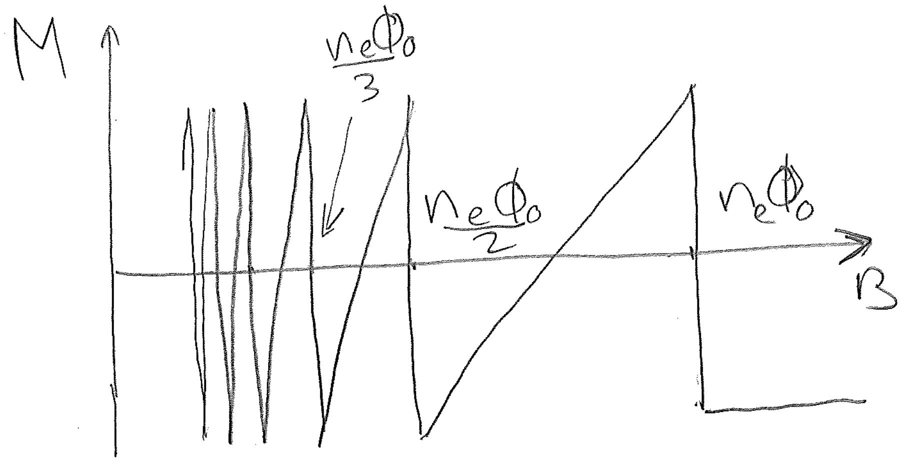
$$3\frac{B}{\Phi_0} > n_e > 2\frac{B}{\Phi_0} \iff \frac{n_e \Phi_0}{3} < B < \frac{n_e \Phi_0}{2}$$

Then level with $n=2$ will be partially filled and

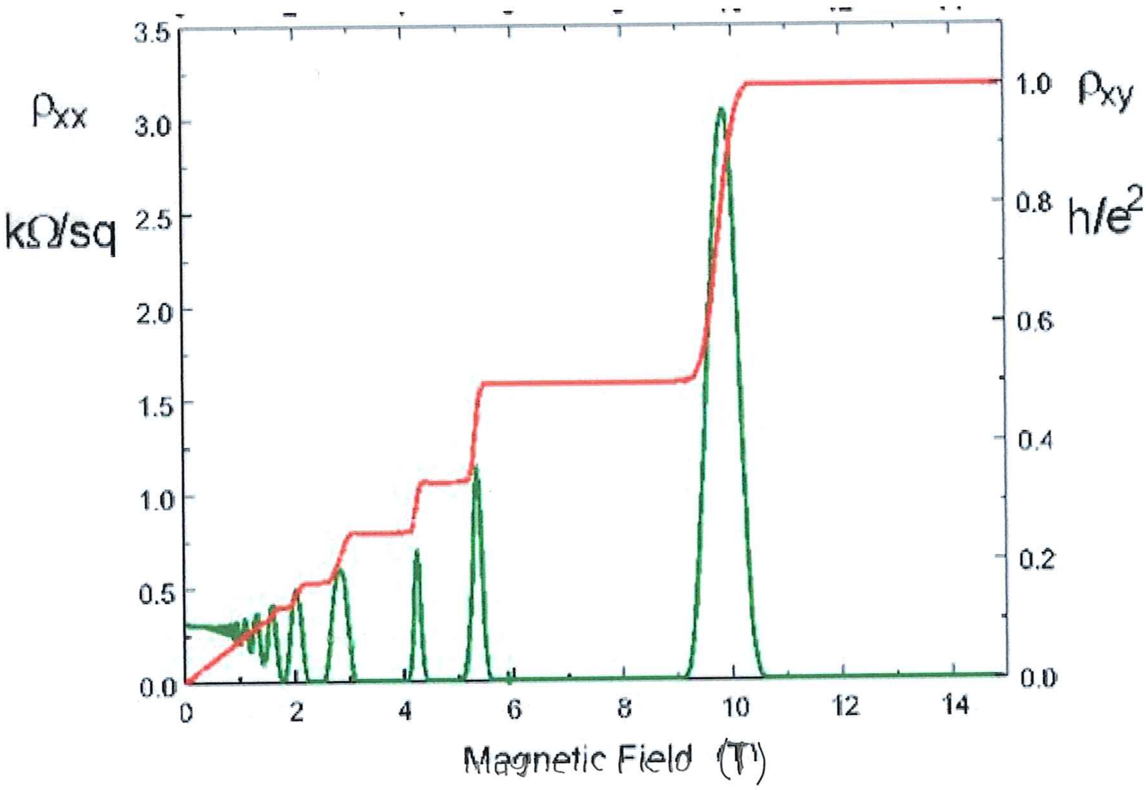
$$\frac{E}{S} = \mu_B B \frac{B}{\Phi_0} + 3\mu_B B \frac{B}{\Phi_0} + \left(n_e - \frac{2B}{\Phi_0}\right) 5\mu_B B =$$

$$= 5\mu_B n_e B - 6\mu_B \frac{B^2}{\Phi_0} \quad \text{and}$$

$$M = -5\mu_B n_e + 12\mu_B \frac{B}{\Phi_0} \quad \text{etc.}$$



Lecture 25) Quantum Hall effect



Integer quantum Hall effect is rather similar to de Haas-van Alphen oscillations

Indeed we can rewrite the Hall conductivity in a high magnetic field as

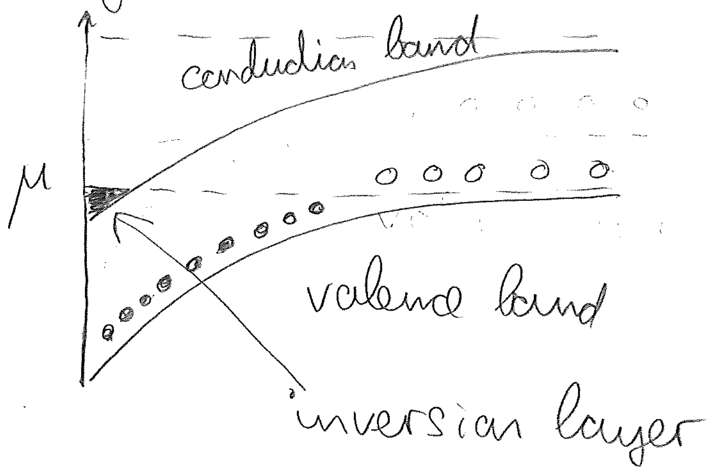
$$\sigma_{xy} = \frac{n_e e c}{B} = \frac{n_e \Phi_0}{B} \frac{e^2}{\hbar} = \nu \frac{e^2}{\hbar}$$

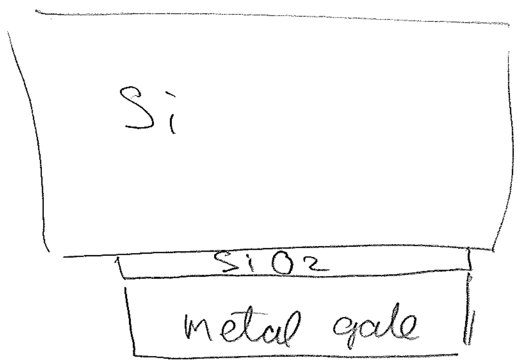
with $\nu = \dfrac{n_e \Phi_0}{B}$ = filling fraction of Landau levels. For integer $\nu = n$ we obtain

$$\sigma_{xy} = n\frac{e^2}{h}, \quad \rho_{xy} = \frac{1}{n}\frac{h}{e^2}$$

Integer quantum Hall effect was observed in 1980 by von Klitzing, Dorda and Pepper

They used Si MOSFET discussed in Lecture 10



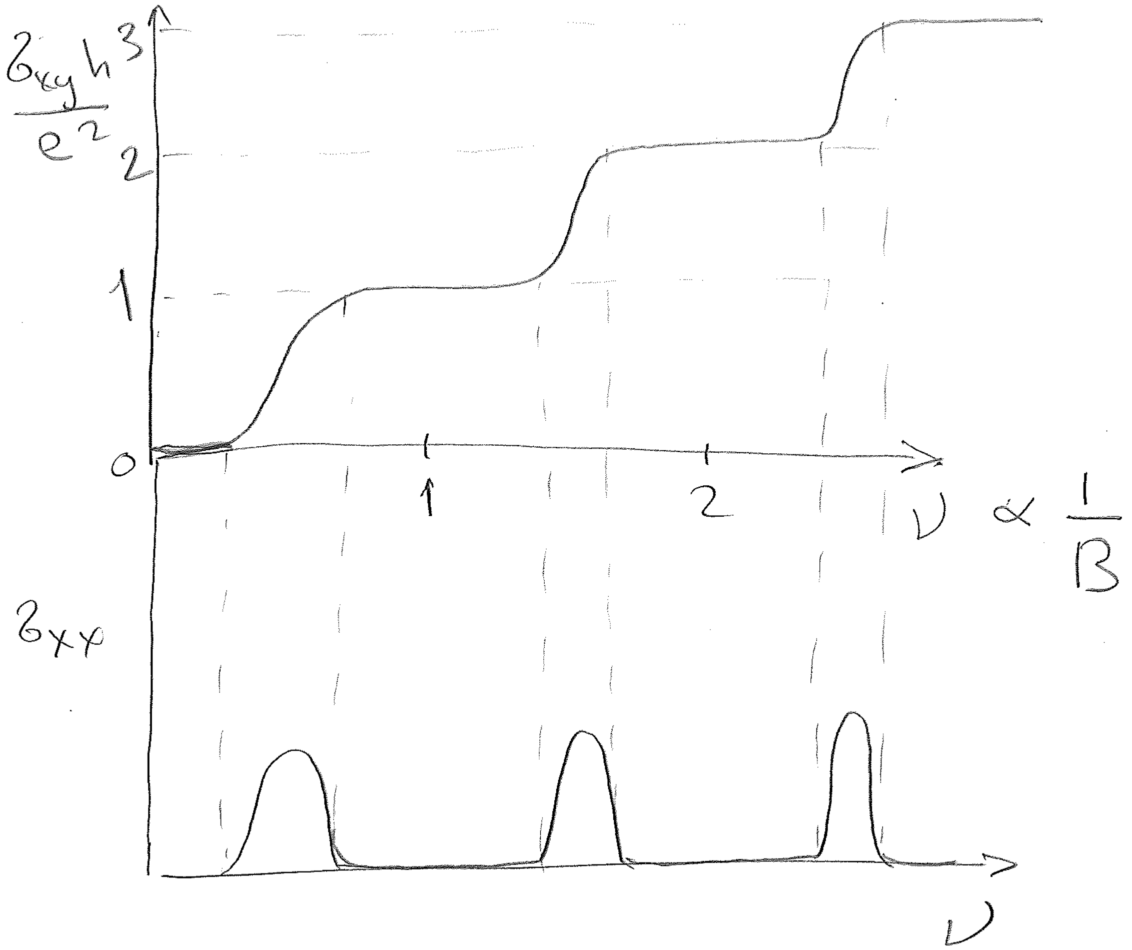


In the inversion layer near the gate a 2-dimensional electron gas 2DEG was formed with high mobility. In high field (1-30T) they observed that the Hall resistivity is quantized $R_{xy} = \frac{1}{n} R_K$, R_K - von Klitzing constant $R_K = 25812.807557\,\Omega$

This constant known with the accuracy better than 10^{-10} is used now as Ohm's standard

In the range where the Hall conductivity shows integer plateaus the longitudinal conductivity vanishes (and the resistivity as well since

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2})$$



In 1982 Tsui, Störmer and Gossard discovered fractional quantum Hall effect that corresponds to certain rational filling factor

Hall effect in 2DEG

As in the previous lecture we choose the Landau gauge $A = (0, Bx, 0)$ then the Schrödinger equation for the wave function

$$\psi(x,y) = e^{i\frac{k_y y}{\hbar}} \psi(x)$$

is

$$-\frac{\hbar^2}{2m}\psi'' + \frac{m\omega_c^2}{2}\left(x - \frac{k_y c}{eB}\right)^2 \psi(x) = \varepsilon\psi$$

With the energy levels given

$$\varepsilon_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

The lowest energy eigenstate has

$$\psi(x,y) = e^{i\frac{k_y y}{\hbar}} \frac{e^{-\frac{(x-x_0)^2}{2\ell^2}}}{\sqrt{2\pi\ell^2}}$$

$\ell = \sqrt{\frac{\hbar c}{|eB|}}$ gives the extension of the wave function. $x_0 = \frac{k_y \ell^2}{\hbar}$

Now let us introduce electric field $E \parallel x$. It adds potential

$$v(r) = -eE_x x$$

This term can be included by shifting the parabola $X_0(k_y) \rightarrow X_0'(k_y) = \frac{k_y \ell^2}{\hbar} + \frac{eE_x}{m\omega_c^2}$

And the lowest Landau level is

$$E_{n=0}(k_y) = \frac{\hbar\omega_c}{2} - \frac{k_y \ell^2 e E_x}{\hbar} - \frac{m}{2}\left(\frac{cE_x}{B}\right)^2$$

The wave function is the same with replacing $X_0(k_y)$ by $X_0'(k_y)$

Since the energy now depends on k_y the velocity of the electrons is

$$v_y(k_y) = \frac{dE(k_y)}{dy} = -\frac{eE_x\ell^2}{\hbar} = -\frac{cE_x}{B}$$

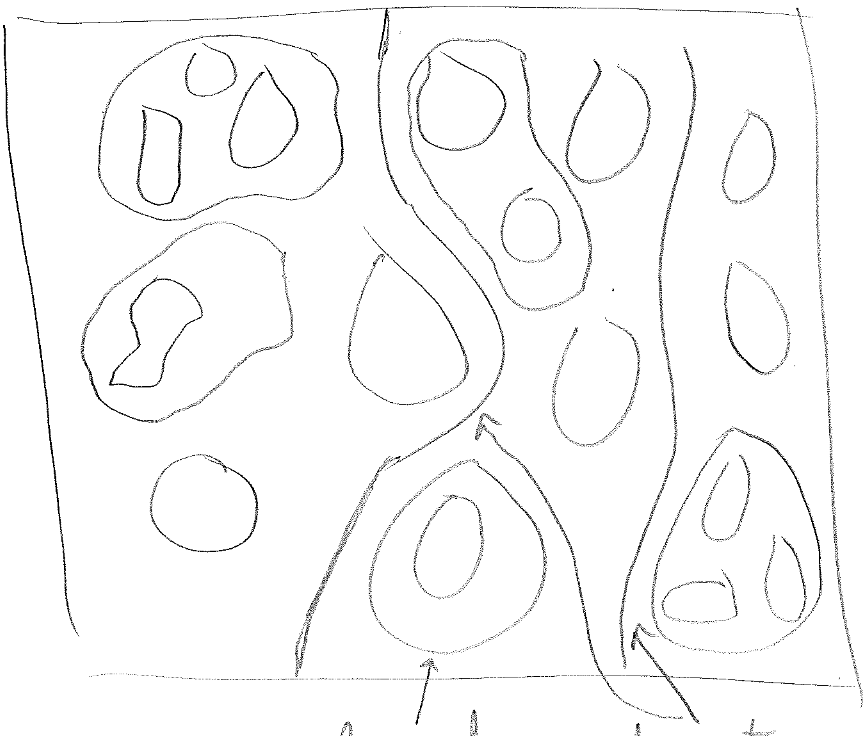
The current density is

$$j = -e n_e v_y(k_y) = e n_e \frac{cE_x}{B} \Rightarrow$$

$$\sigma_{xy} = \frac{n_e e c}{B} = \nu \frac{e^2}{h}$$

which agrees with the classical result

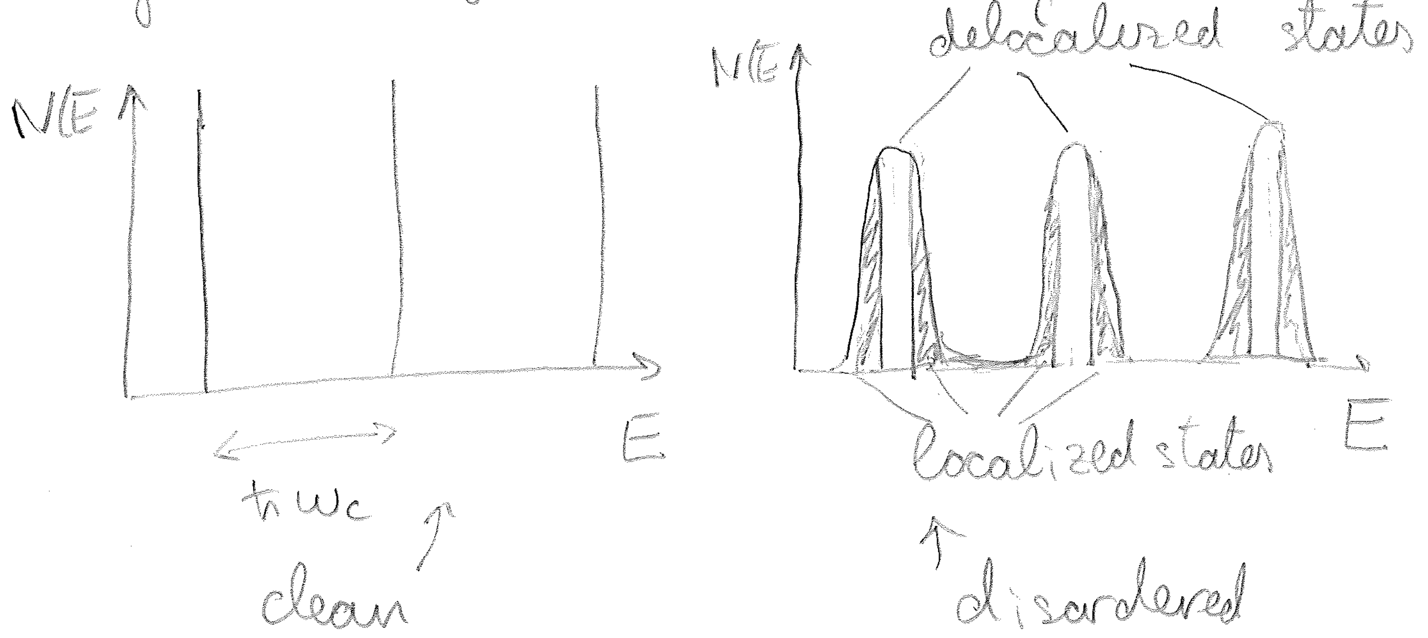
The plateaus in the Hall conductivity correspond to $\nu_n \in \mathbb{N}$ $\Rightarrow$ complete filling of n Landau levels. The longitudinal conductivity vanishes. This is due to localization. In high magnetic field electron motion is along the equipotential trajectories that are either filled or empty



There are closed and extended trajectories

For weak disorder Landau levels are broadened. Lowest energy states correspond to the closed orbits around the minima of the random potential

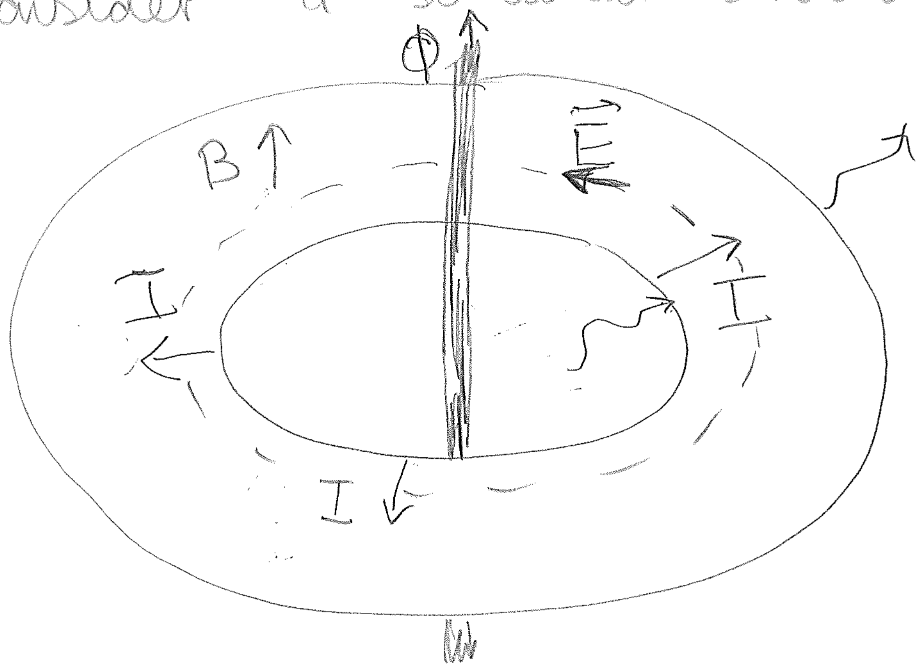
For high energies we have closed orbits near the maxima. In the intermediate energy range we get extended trajectories



Changing magnetic field we change the position of the Fermi level with respect to the Landau levels. When the Fermi level is inside the mobility gap we have $\sigma_{xx} = 0$ and integer number of the Landau level filled — plateau. When the Fermi level is inside the extended states we have transition between plateaus and appearence of σ_{xx} and ρ_{xx}

Laughlin's gauge arguments

Consider a so called Corbino disc geometry



We apply current from inside to outside
There is constant external magnetic field B
In Laughlin's gedanken experiment one introduces magnetic flux Φ through the hole which is adiabatically varied
This flux influences only extended trajectories
it introduces Aharonov Bohm phase to the wave function

$$\psi \to \psi e^{i\frac{e\int A\,dx}{c}} = \psi e^{i\frac{\delta\Phi}{\Phi_0}\varphi}$$

Since we have quantization of orbits

$$\Phi_m = (m+\gamma)\Phi_0$$

changing flux by flux quantum Φ_0 will change $m \to m \pm 1$.

With $\delta\Phi = \Phi_0$ there is no change in the states of the system. But electron is transfered from one orbit to the next.

Induced electric field $\oint \vec{E}.d\vec{\ell} = E_\varphi 2\pi r = -\frac{1}{c}\frac{\partial\Phi}{\partial t}$

Transported charge $\Delta Q = \int dt\, j_r 2\pi r = \int dt\, \sigma_H E_\varphi 2\pi r =$

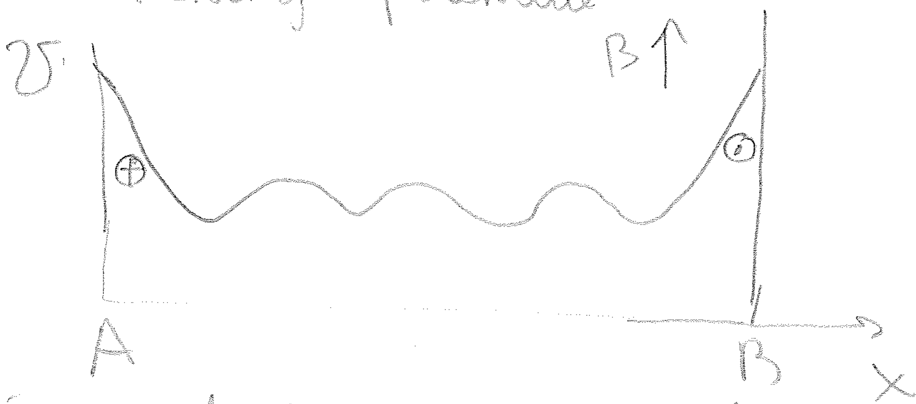
$$= -\sigma_{xy}\frac{\Phi_0}{c}$$

But the transfered charge is quantized

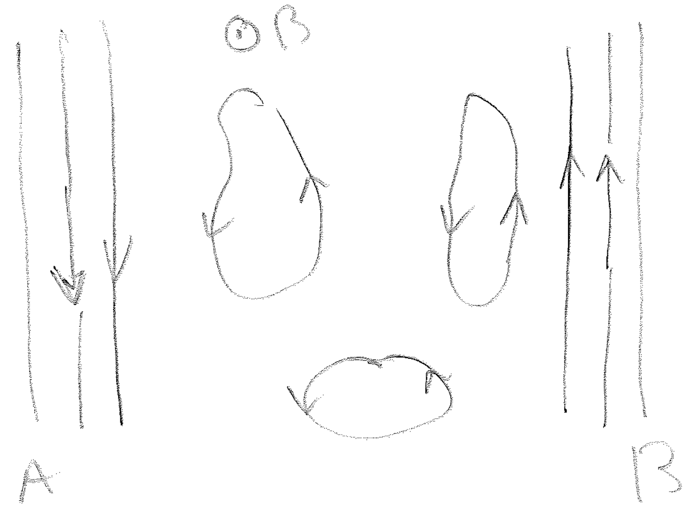
$$\Delta Q = -ne \Rightarrow \sigma_{xy} = \frac{ec}{\Phi_0} n = n\frac{e^2}{h}$$

Edge states

Near the edge of the 2 DEG there is a confining potential



This potential is similar to electric field discussed on page 5. This will give the extended trajectory that percolates along the edge. These states propagate in the opposite directions on the opposite edges



chiral edge states

The current along the edge is $\sum e v_y$

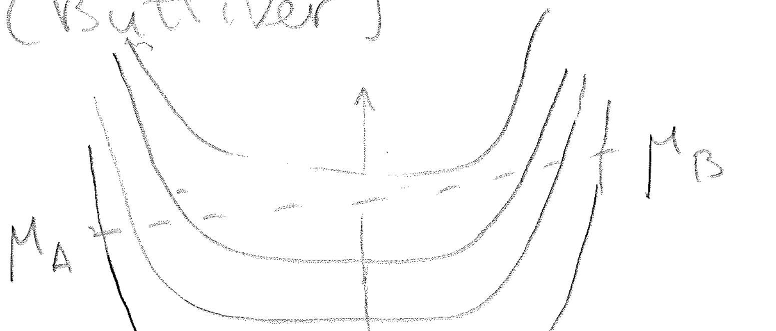
$$I = e \int \frac{dk_y}{2\pi\hbar} \frac{dE_n(k_y)}{dk_y} = \frac{e}{h}\left(\mu - E_n^0\right)$$

If $\mu_A - \mu_B = e V_H$ then the total current from both edges is

$$I = I_A - I_B = \frac{e}{h}\left(\mu_A - \mu_B\right) = \frac{e^2}{h} V_H$$

and $\sigma_{xy} = \frac{e^2}{h}$

Here we assum that all the states in the bulk are localized. General picture (Buttiker)



Every Landau level gives one edge state, thus $\sigma_H = n \frac{e^2}{h}$