Solid State Theory
Exercise 8

Problem 8.1 Lindhard function

In the lecture it was shown how to derive the dynamical linear response function $\chi_0(\vec{q},\omega)$ which is also known as the Lindhard function:

$$\chi_0(\vec{q},\omega) = \frac{1}{\Omega}\sum_{\vec{k}} \frac{n_{\vec{k}+\vec{q}} - n_{\vec{k}}}{\epsilon_{\vec{k}+\vec{q}} - \epsilon_{\vec{k}} - \hbar\omega - i\hbar\eta}. \tag{1}$$

Calculate the static Lindhard function $\chi_0(\vec{q},\omega = 0)$ of free electrons for the (a) 1- and (b) 3-dimensional cases at $T = 0$. Show that $\chi_0(\vec{q},\omega = 0)$ has a vanishing imaginary part.
Hint: For the calculation of the real part of $\chi_0(\vec{q},\omega)$ you may use the equation $\lim_{\eta\to 0}(z - i\eta)^{-1} = \mathcal{P}\,(1/z) + i\pi\delta(z)$. Note that in 3 dimensions we can choose $\vec{q} = q\vec{e}_z$ to point in the z-direction due to the isotropy of a system of free electrons. Also, changing to cylindrical coordinates in order to calculate the integral turns out to be helpful.