Dimensionless Ginzburg - Landau equations:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}\left[\frac{\hbar\vec{\nabla}}{i} - q\vec{A}\right]^2\psi = 0 \quad \text{— } (1)$$

Scale $\psi \to \Psi\sqrt{\frac{|\alpha|}{\beta}}$, then (1) becomes

$$\alpha\Psi + |\alpha|\,|\Psi|^2\Psi + \frac{1}{2m}\left[\frac{\hbar\vec{\nabla}}{i} - q\vec{A}\right]^2\Psi = 0 \quad \text{— } (2)$$

Dividing by $|\alpha|$ and for $T < T_c$, (2) becomes

$$-\Psi + |\Psi|^2\Psi + \frac{1}{2m|\alpha|}\left[\frac{\hbar\vec{\nabla}}{i} - q\vec{A}\right]^2\Psi = 0 \quad \text{— } (3)$$

Now choose $\vec{y} = \frac{\vec{x}}{\lambda_L}$ and $\vec{a} = \frac{\vec{A}}{\sqrt{2}\lambda_L B_c}$,

then

$$-\Psi + |\Psi|^2\Psi + \left[\frac{\vec{\nabla}_y}{\kappa i} - \vec{a}\right]^2\Psi = 0 \quad \rightarrow (4)$$

where $\kappa = \frac{\lambda_L}{\xi} = \sqrt{\frac{2m\beta}{\hbar^2 q^2\mu_0}}$.

We use $\frac{B_c^2}{\mu_0} = \frac{\alpha^2}{\beta}$

$$\lambda_L^2 = \frac{m\beta}{q^2\mu_0|\alpha|}$$

$$B_c^2 = \mu_0\frac{\alpha^2}{\beta}.$$

Scaling for $\vec{A}$ implies

$$\vec{b} = \frac{\vec{B}}{\sqrt{2}B_c}.$$

Note: B_c is a critical field obtained from thermodynamics!

Proximity effect

Consider a normal metal - SC interface in one dimension

$N(T_{cN})$ — $SC(T_{cS})$ Temperature is chosen such that

$$T_{cN} < T < T_{cS}$$

Using Eq. (3) here for the SC region and assuming Ψ to be real, we have,

$$-\xi^2 \frac{d^2\Psi}{dx^2} - \Psi + \Psi^3 = 0 \quad \text{(a)}$$ Integrating w.r.t. Ψ

$$-\frac{\xi^2}{2}\left(\frac{d\Psi}{dx}\right)^2 - \frac{\Psi^2}{2} + \frac{\Psi^4}{4} = C' \quad \text{(b)}$$ But as $x \to \infty$ $\Psi \to 1$, $\frac{d\Psi}{dx} \to 0$.

$\Rightarrow C' = -\frac{1}{2}$. Soln. to eqn (b) is

$$\Psi = \tanh \frac{x - x_0}{\sqrt{2}\,\xi}$$

x_0 determined by boundary condition

$$\frac{1}{\Psi}\frac{d\Psi}{dx}\bigg|_{x=0} = \frac{1}{b}$$

We find

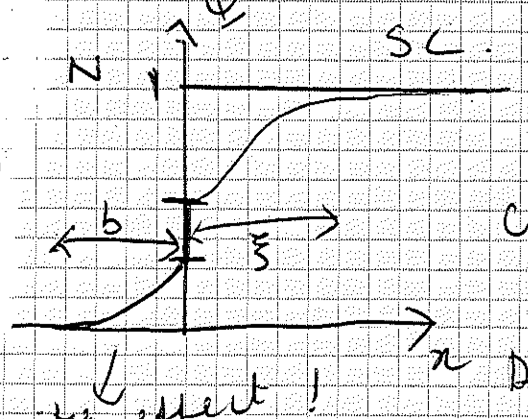
$$\sinh \sqrt{2}\,\frac{x_0}{\xi} = \sqrt{2}\,\frac{b}{\xi}.$$

b is lengthscale over which order parameter penetrates N region

In the Normal metal region, since order parameter Ψ is very small, we have.

$$-\xi_N^2 \frac{d^2\Psi}{dx^2} + \Psi = 0 \quad \text{with} \quad \Psi(-\infty) \to 0.$$

$$\Longrightarrow \Psi(x) \propto \exp \frac{x}{\xi_N} \quad \text{in this region.}$$



Proximity effect!
Order parameter penetrates a boundary layer.

Sometimes $b = \xi_N$, but not always!. $\xi_N \sim 100\,\text{nm} - 1\,\mu\text{m}$.

Clean metals $\xi_N \approx \frac{\hbar v_{FN}}{2\pi k_B T}$

Dirty metals $\xi_N \sim \sqrt{\frac{\hbar v_{FN} l_N}{6\pi k_B T}}$

v_{FN} — Fermi velocity
l_N — mean free path