$$\mathcal{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

$$\mathcal{H}|n\rangle = \epsilon_n|n\rangle$$
$$\epsilon_n = \hbar\omega(n + 1/2)$$

raising $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ create particle with $E = \hbar\omega$

lowering $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ delete particle with $E = \hbar\omega$

2nd quantization: $[\hat{a}, \hat{a}^\dagger] = 1 \quad [\hat{a}, \hat{a}] = [\hat{a}^\dagger, \hat{a}^\dagger] = 0$

$\hat{a}^\dagger\hat{a} = \hat{n}$ particle number operator

$$\langle\hat{n}\rangle = \frac{1}{Z}\sum_{n=0}^{\infty} n e^{-\beta\epsilon_n} = \frac{1}{e^{\beta\hbar\omega} - 1}$$ "bosonic"

$$\mathcal{H} = \frac{1}{2}\left(P^2 + \omega^2 Q^2\right) \xrightarrow{\text{quantization}} [\hat{Q}, \hat{P}] = i\hbar \quad \begin{cases} \hat{Q} = \sqrt{\dfrac{\hbar}{2\omega}}(\hat{a} + \hat{a}^\dagger) \\ \hat{P} = i\omega\sqrt{\dfrac{\hbar}{2\omega}}(\hat{a} - \hat{a}^\dagger) \end{cases}$$

$$\mathcal{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)$$

$$\mathcal{H}|n\rangle = \epsilon_n|n\rangle$$
$$\epsilon_n = \hbar\omega(n + 1/2)$$

canonical ensemble

$$Z = tre^{-\beta\mathcal{H}} = e^{-\beta\hbar\omega/2}\sum_{n=0}^{\infty} e^{-\beta\hbar\omega n} = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}}$$

internal energy $\quad U = -\dfrac{\partial \ln Z}{\partial\beta} = \dfrac{1}{2}\hbar\omega + \dfrac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$