Electromagnetic wave - harmonic oscillator

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\vec{A} = 0 \qquad \vec{\nabla}\cdot\vec{A} = 0$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

plane waves in cavity (*L x L x L*):

$$\vec{A}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\lambda}\left\{A_{\vec{k}\lambda}\vec{e}_{\vec{k}\lambda}e^{i\vec{k}\cdot\vec{r}-i\omega t} + A^*_{\vec{k}\lambda}\vec{e}^{\,*}_{\vec{k}\lambda}e^{-i\vec{k}\cdot\vec{r}+i\omega t}\right\}$$

$$\omega = \omega_{\vec{k}} = c|\vec{k}| \qquad \vec{e}_{\vec{k}\lambda}\cdot\vec{k} = 0$$

Periodic boundary conditions:

$$\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z) \qquad n_i = 0, \pm 1, \pm 2, \ldots$$

Electromagnetic wave - Canoncial quantization

$$Q_{\vec{k}\lambda} = \frac{1}{\sqrt{4\pi c}}\left(A_{\vec{k}\lambda} + A^*_{\vec{k}\lambda}\right) \qquad P_{\vec{k}\lambda} = \frac{i\omega_{\vec{k}}}{\sqrt{4\pi c}}\left(A_{\vec{k}\lambda} - A^*_{\vec{k}\lambda}\right)$$

Hamiltonian: $$\mathcal{H} = \int d^3r \frac{\vec{E}^2 + \vec{B}^2}{8\pi} = \sum_{\vec{k},\lambda} \frac{\omega_{\vec{k}}}{2\pi c}\left|A_{\vec{k}\lambda}\right|^2 = \frac{1}{2}\sum_{\vec{k},\lambda}\left(P^2_{\vec{k}\lambda} + \omega^2_{\vec{k}} Q^2_{\vec{k}\lambda}\right)$$

canonical quantization $$[Q_{\vec{k},\lambda}, P_{\vec{k}',\lambda'}] = i\hbar\delta_{\vec{k}\vec{k}'}\delta_{\lambda\lambda'}$$

raising / lowering operators

$$A^*_{\vec{k}\lambda} \to a^\dagger_{\vec{k}\lambda}$$

$$A_{\vec{k}\lambda} \to a_{\vec{k}\lambda}$$

$$[a_{\vec{k}\lambda}, a^\dagger_{\vec{k}'\lambda'}] = \delta_{\vec{k}\vec{k}'}\delta_{\lambda\lambda'}$$

$$\mathcal{H} = \sum_{\vec{k},\lambda} \hbar\omega_{\vec{k}}\left(a^\dagger_{\vec{k}\lambda} a_{\vec{k}\lambda} + \frac{1}{2}\right) = \sum_{\vec{k},\lambda} \hbar\omega_{\vec{k}}\left(n_{\vec{k}\lambda} + \frac{1}{2}\right)$$

create bosonic particles in mode

$(\vec{k}, \lambda)$ with $\omega_{\vec{k}}$ photon