

Not part of the exam

# Contents

<b>1</b>	<b>Kinetic approach to statistical physics</b>	<b>5</b>
1.1	Time evolution and Master equation . . . . .	5
1.1.1	$H$ -function and information . . . . .	8
1.1.2	Simulation of a two-state system . . . . .	9
1.1.3	Equilibrium of a system . . . . .	11
1.2	Analysis of a closed system . . . . .	12
1.2.1	$H$ and the equilibrium thermodynamics . . . . .	12
1.2.2	Master equation . . . . .	13
1.2.3	Irreversible processes and increase of entropy . . . . .	14
<b>2</b>	<b>Classical statistical Physics</b>	<b>18</b>
2.1	Gibbsian concept of ensembles . . . . .	18
2.1.1	Liouville Theorem . . . . .	19
2.1.2	Equilibrium system . . . . .	20
2.2	Microcanonical ensemble . . . . .	20
2.2.1	Entropy . . . . .	21
2.2.2	Relation to thermodynamics . . . . .	23
2.2.3	Ideal gas - microcanonical treatment . . . . .	23
2.3	Canonical ensemble . . . . .	25
2.3.1	Thermodynamics . . . . .	26
2.3.2	Equipartition law . . . . .	27
2.3.3	Ideal gas - canonical treatment . . . . .	28
2.4	Grand canonical ensemble . . . . .	28
2.4.1	Relation to thermodynamics . . . . .	30
2.4.2	Ideal gas - grand canonical treatment . . . . .	30
2.4.3	Chemical potential in an external field . . . . .	31
2.5	Fluctuations . . . . .	32
2.5.1	Energy . . . . .	32
2.5.2	Particle number . . . . .	33
2.5.3	Magnetization . . . . .	34
<b>3</b>	<b>Quantum Statistical Physics</b>	<b>35</b>
3.1	Basis of quantum statistical physics . . . . .	35
3.2	Density matrix . . . . .	37
3.3	Ensembles in quantum statistics . . . . .	38
3.3.1	Microcanonical ensemble . . . . .	38
3.3.2	Canonical ensemble . . . . .	39
3.3.3	Grand canonical ensemble . . . . .	39
3.4	Ideal quantum paramagnet - canonical ensemble . . . . .	40
3.4.1	Spin 1/2 . . . . .	40
3.4.2	Spin $S$ - classical limit . . . . .	41
3.5	Ideal quantum gas - grand canonical ensemble . . . . .	43

3.6	Properties of Fermi gas . . . . .	45
3.6.1	High-temperature and low-density limit . . . . .	46
3.6.2	Low-temperature and high-density limit: degenerate Fermi gas . . . . .	47
3.6.3	Spin-1/2 Fermions in a magnetic field . . . . .	48
3.7	Bose gas . . . . .	50
3.7.1	Bosonic atoms . . . . .	50
3.7.2	High-temperature and low-density limit . . . . .	51
3.7.3	Low-temperature and high-density limit: Bose-Einstein condensation . . . . .	51
3.8	Photons and phonons . . . . .	54
3.8.1	Blackbody radiation - photons . . . . .	56
3.8.2	Phonons in a solid . . . . .	58
3.9	Diatomic molecules . . . . .	60
<b>4</b>	<b>Identical Quantum Particles - Formalism of Second Quantization</b>	<b>64</b>
4.1	Many-body wave functions and particle statistics . . . . .	64
4.2	Independent, indistinguishable particles . . . . .	65
4.3	Second Quantization Formalism . . . . .	66
4.3.1	Creation- and annihilation operators . . . . .	66
4.3.2	Field operators . . . . .	68
4.4	Observables in second quantization . . . . .	69
4.5	Equation of motion . . . . .	70
4.6	Correlation functions . . . . .	72
4.6.1	Fermions . . . . .	72
4.6.2	Bosons . . . . .	74
4.7	Selected applications . . . . .	75
4.7.1	Spin susceptibility . . . . .	75
4.7.2	Bose-Einstein condensate and coherent states . . . . .	77
4.7.3	Phonons in an elastic medium . . . . .	81
<b>5</b>	<b>One-dimensional systems of interacting degrees of freedom</b>	<b>84</b>
5.1	Classical spin chain . . . . .	84
5.1.1	Thermodynamics . . . . .	84
5.1.2	Correlation function . . . . .	85
5.1.3	Susceptibility . . . . .	87
5.2	Interacting lattice gas . . . . .	88
5.2.1	Transfer matrix method . . . . .	88
5.2.2	Correlation function . . . . .	90
5.3	Long-range order versus disorder . . . . .	91
<b>6</b>	<b>Phase transitions</b>	<b>93</b>
6.1	Ehrenfest classification of phase transitions . . . . .	93
6.2	Phase transition in the Ising model . . . . .	95
6.2.1	Mean field approximation . . . . .	95
6.2.2	Instability of the paramagnetic phase . . . . .	97
6.2.3	Phase diagram . . . . .	99
6.3	Gaussian transformation . . . . .	101
6.3.1	Correlation function and susceptibility . . . . .	103
6.4	Ginzburg-Landau theory . . . . .	105
6.4.1	Ginzburg-Landau theory for the Ising model . . . . .	106
6.4.2	Critical exponents . . . . .	107
6.4.3	Range of validity of the mean field theory - Ginzburg criterion . . . . .	108
6.5	Self-consistent field approximation . . . . .	109
6.5.1	Renormalization of the critical temperature . . . . .	110

6.5.2	Renormalized critical exponents . . . . .	112
6.6	Long-range order - Peierls' argument . . . . .	113
6.6.1	Absence of finite-temperature phase transition in the 1D Ising model . . .	113
6.6.2	Long-range order in the 2D Ising model . . . . .	114
<b>7</b>	<b>Superfluidity</b>	<b>116</b>
7.1	Quantum liquid Helium . . . . .	116
7.1.1	Superfluid phase . . . . .	117
7.1.2	Collective excitations - Bogolyubov theory . . . . .	119
7.1.3	Gross-Pitaevskii equations . . . . .	124
7.2	Berezinskii-Kosterlitz-Thouless transition . . . . .	127
7.2.1	Correlation function . . . . .	127
7.2.2	Topological excitations and BKT transition . . . . .	128