Introduction to Superconductivity
Exercise Sheet 8

HS14
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Discussion date: 19 November 2014

Short Josephson junctions

Josephson junctions are classified as either lumped or extended, depending on whether the spatial extension of the contact is taken into account or not, i.e., whether the phase and current vary across the junction. For example, the circuit elements in the RCSJ model are basic lumped junctions.

Extended Josephson junctions in turn are classified as either short or long, depending on whether their spatial dimension is smaller or bigger than the Josephson penetration depth*, i.e., whether self-fields of the junction can be neglected or are important.*

Exercise 1: Relation between phase difference and field.

We consider the following setup, illustrated in Fig. 1. Two superconductors, extending infinitely in x-direction and having width w in z-direction, are connected by a weak link of dimension d in y-direction. A magnetic field of strength B_0 is applied through the gap in x-direction.

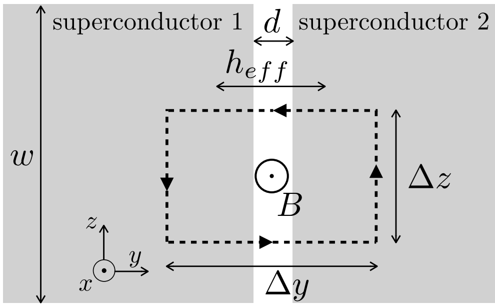

Figure 1: Two superconductors with a weak link of extension d.

(a) Sketch the profile of the magnetic field, $B(y)\hat{x}$.

In which direction does the vector potential $\boldsymbol{A}$ point?

Calculate the flux as $\Phi(B_0, d, \lambda_1, \lambda_2, \Delta z)$, where λ_i are the London penetration depths of the two superconductors.

What is the effective length of the gap, h_{eff}, also called the magnetic thickness?

(b) Find the relation between the phase difference $\phi(z)$ of the two superconductors and the flux:

Integrate the phase gradient $\nabla\theta$ along a closed path perpendicular to the field (dashed line in Fig. 1), where we take Δy to be much larger than h_{eff} and Δz infinitesimal. Split the integral into convenient segments. To evaluate the integrals, use the relation from Exercise Sheet 6, Eq. (1), and the gauge invariant phase difference, where appropriate.

Comment: The gauge invariant phase difference across a junction is

$$\phi = \theta_2 - \theta_1 - \frac{2\pi}{\Phi_0} \int_1^2 \boldsymbol{A} d\boldsymbol{l}. \tag{1}$$

(see lecture notes on the SQUID)

(c) Combine your results to find the relation between the phase difference and the applied field.

Can you also formulate a general relation, independent of the geometry of the actual setup?

Exercise 2: Current distribution and maximal current.

We still consider the above setup, illustrated in Fig. 1.

In the first exercise, we have found a relation between the phase difference (mod 2π) and the field

$$\frac{\partial \phi}{\partial z} = \frac{2\pi h_{eff}}{\Phi_0} B_0. \tag{2}$$

From the lecture, you know the relation between the current and the phase

$$J_s(z) = J_c \sin\phi, \tag{3}$$

where we assume a constant critical current J_c.

We can now combine these relations to find the maximal current as a function of the applied field, and to find the current distribution inside the junction.

(a) What is the current distribution J_s as a function of the applied field?

Comment: You can do a straightforward integration of the first relation (2), but pay attention to the integration constant.

Sketch the current distribution for different convenient values of $\Phi_J = B_0 h_{eff} w$.

(b) What is the maximal total current that can pass through the junction, as a function of the applied magnetic field?

Make a sketch, do you recognize the pattern?