Phase Transitions and Critical Phenomena

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Exercise Sheet 8

HS 14
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Problem 1 Vortices in 2D XY-model

Energy of a vortex in 2D XY-model depends on its vorticity. In the lecture we derived that if the vorticity is $n = \pm 1$, the energy of the vortex is

$$E = \pi J \log\left(\frac{R}{a}\right) \tag{1}$$

where R is the system size and a is the vortex core radius.

Find the energy of a vortex for a general value of vorticity n.