Phase Transitions and Critical Phenomena

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Exercise Sheet 6

HS 14
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Problem 1 Correlation length of 1D Ising model

In the lecture we used the renormalization group method to treat 1D Ising model. We considered a block spin transformation which mapped a block of 3 neighboring spins onto a single spin of a rescaled system. We showed that such transformation changes the reduced coupling constant $\tilde{J} = J/T$ as

$$\tilde{J}' = \tanh^{-1}\left(\tanh \tilde{J}\right)^3. \tag{1}$$

Furthermore, such transformation rescales the correlation length as

$$\xi(J') = \frac{\xi(J)}{3}. \tag{2}$$

Use these two results to find how the correlation length depends on temperature for $T \to 0$.