Phase Transitions and Critical Phenomena

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Exercise Sheet 2

HS 14
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Problem 1 Landau theory vs. mean field theory

In the lecture we treated the Ising model in two different ways. First, we showed that within the mean field approximation the free energy per site of a system with magnetization M in an external magnetic field H is

$$f = \frac{JM^2}{2} - k_\mathrm{B}T \log\cosh\left(\frac{JM+H}{k_\mathrm{B}T}\right). \tag{1}$$

Second, we argued that close to the critical point the free energy can be expanded as

$$f_\mathrm{L} = atM^2 + \frac{b}{2}M^4 - HM \tag{2}$$

where $t = \frac{T}{T_\mathrm{c}} - 1$.

Show that the Taylor expansion of (1) around the critical point is in fact equivalent to expression (2). What are the corresponding values of a and b?

Problem 2 Tricritical point

Consider the Landau free energy

$$F = a\eta^2 + b\eta^4 + c\eta^6 \tag{3}$$

and study its phase diagram and phase transitions as a function of a, b and c.

Note that the phase diagram contains a point where a line of first-order transition meets a line of second-order transition called a *tricritical point* (TP). At what parameters $a_\mathrm{TP}, b_\mathrm{TP}, c_\mathrm{TP}$ does it occur?