# General relativity. Problem set 11.

*HS 14* *Due:* Tue, December 2, 2014

## 1. The magnitude-redshift relation

In Minkowski spacetime the energy flux $f$ of light coming from a source of intensity $L$ at a fixed distance $d$ from the observer is

$$f = \frac{L}{4\pi d^2} \,.$$

i) Show that the corresponding relation in a Friedmann universe is

$$f = \frac{L}{4\pi d^2 (1+z)^2} \Big(\frac{\chi}{\mathrm{sinn}\chi}\Big)^2 = \frac{L}{4\pi d^2 (1+z)^2} \Big(1 - \frac{\Omega_k}{3}(Hd)^2 + \ldots\Big) \,, \tag{1}$$

where $d$ is the proper distance, i.e., the present distance between the source (at $\chi = 0$) and the observer (at $\chi$).

*Hints:* Eq. (1) is purely classical, yet to derive it it is convenient to conceive light as a stream of photons. The energies of the photons relative to sender and receiver are different because of redshift. The proper times between successive photons are too. The area over which the light is spread is $A = 4\pi(R_0 \,\mathrm{sinn}\,\chi)^2$, $(a(t_0) = 1)$.

ii) Combine (1) with the distance-redshift relation on p. 60 of the lecture notes to conclude

$$f = \frac{LH^2}{4\pi z^2}\big(1 - (1-q)z + O(z^2)\big) \,.$$

*Remark:* the apparent magnitude is essentially $-\log f$.

## 2. Killing vectors

Find explicit expressions for a complete set of (not necessarily time-like) Killing vector fields for the following spaces (i, ii):

i) Minkowski spacetime, with metric $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$.

*Hint:* Show that $K_{\mu,\nu\sigma} = 0$, whence $K_\mu = a_\mu + {b_\mu}^\nu x_\nu$.

ii) A spacetime with coordinates $\{u, v, x, y\}$ and metric

$$ds^2 = (du\,dv + dv\,du) - a^2(u)dx^2 - b^2(u)dy^2$$

where $a$ and $b$ are arbitrary functions of $u$. This represents a spacetime carrying a gravitational wave.

*Hint:* There are five linearly independent Killing fields, out of which three are obvious. Two more may be found by the ansatz

$$K^\mu = (0, f, g, h) \tag{2}$$

with $f, g, h$ functions of $u$ (but not of $v$), and of either $x$ or $y$. It helps to compute the Christoffel symbols first.