General relativity. Problem set 7.

HS 14 *Due:* Tue, November 4, 2014

1. An ideal fluid

Consider an ideal fluid made of particles of mass m. Suppose that in its rest frame they have number density n and isotropically distributed velocities of fixed length v, i.e. $\vec{v} = v\vec{e}$ with $\vec{e}$ uniformly distributed on the unit sphere. Compute the energy density ρc^2 and the pressure p. What happens in the limits

1. $v \to 0$,
2. $v \to c$ and $m \to 0$, with $mc^2(1-(v/c)^2)^{-1/2} \to E$ (photons)?

In particular, compare $T^\mu{}_\mu$ with the trace of the electromagnetic energy-momentum tensor (4.10).

2. A variational principle

Consider the variational principle, stated on p. 34, describing the motion $x(\tau)$ of a particle (charge e) falling through an electromagnetic field of potential $A^\mu(x)$:

$$\delta \int_{(1)}^{(2)} d\tau \big(c^2 + \frac{e}{mc}(\dot{x}, A)\big) = 0$$

with fixed endpoints (1) and (2) in space and time. Here $\dot{x} = dx/d\tau$ with τ being proper time. Find the corresponding equation of motion.

Hint: The endpoints of τ are not fixed. Thus, parametrize the trajectory by another parameter, which remains fixed under variations.