Exercise 1. ***Oblique Corrections***

In this exercise we want to investigate the so-called oblique corrections to electro-weak interactions. These corrections were introduced to analyse new physics phenomena for processes of $e^+e- \to q\bar{q}$ as measured at LEP. In the Standard Model fermionic currents $J^\mu_Q(p^2)$ are coupled to the $SU_I(2) \times U_Y(1)$ gauge fields via the interaction Lagrangian

$$\mathcal{L} = gW^+_\mu J^\mu_+ + gW^-_\mu J^\mu_- + gZ_\mu J^\mu_Z + eA_\mu J^\mu_{EM},$$

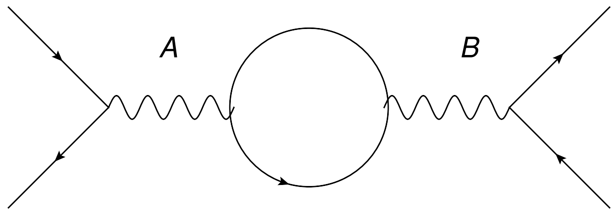
where

$$J^\mu_Q = i\bar{\psi}\gamma^\mu Q\psi.$$

The different charges for the interactions are given by

$$Q_\pm = \frac{1}{\sqrt{2}}T^\pm P_L, \qquad Q_Z = \frac{1}{c_W}\left(T^3 - s^2_W Q\right)P_L, \qquad Q_{EM} = eQ.$$

We want to consider one-loop corrections to resonant $q\bar{q}$ production in lepton collisions. Some leading corrections are given by fermion loops inserted into the gauge boson propagator. The gauge bosons A and B can be $W^\pm, Z$ or γ.



We can decompose the resonant propagator corrections as

$$\Pi^{\mu\nu}_{AB}(p) = g^{\mu\nu}\Pi_{AB}(p^2) + \frac{p^\mu p^\nu}{p^2}\Delta_{AB}(p^2).$$

(a) If we are interested in processes as described above why can we restrict ourselves to terms proportional to $g^{\mu\nu}$?

(b) Calculated $\Pi_{AB}(p^2)$ for two left-handed $\Pi_{LL}(p^2)$ and one left- and one right-handed $\Pi_{LR}(p^2)$ current with two different fermion masses in the loop. Use dimensional regularisation in combination with anti-commuting γ^5 matrices. To compute the loop integrals introduce Feynman parameters and express your result in terms of the following integral definitions. Compute up to finite order in the dimensional regulator $d = 4 - 2\epsilon$.

$$b_0(p^2, m_1, m_2) = \int_0^1 dx \log\left(xm_1^2 + (1-x)m_2^2 - p^2x(1-x)\right), \tag{1}$$

$$b_1(p^2, m_1, m_2) = \int_0^1 dxx \log\left(xm_1^2 + (1-x)m_2^2 - p^2x(1-x)\right), \tag{2}$$

$$b_2(p^2, m_1, m_2) = \int_0^1 dxx(1-x) \log\left(xm_1^2 + (1-x)m_2^2 - p^2x(1-x)\right). \tag{3}$$

(c) Use your result from above to compute the correction to the photon propagator $e^2\Pi_{QQ}(p^2)$. You should find that your corrections vanish as $p^2 \to 0$. Can you make a connection between the propagator correction of the photon and its renormalisation counter-terms? Why does the correction have to vanish as p^2 goes to zero?

In the lecture you introduced the Peskin-Takeuchi parameters S and T. These are related to the propagator corrections as

$$
\begin{aligned}
S &\sim 4e^2 \left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right], &(4)\\
T &\sim \frac{e^2}{s_W^2 c_W^2 M_Z^2} \left[\Pi_{++}(0) - \Pi_{33}(0)\right]. &(5)
\end{aligned}
$$

(d) Compute the leading corrections to the T parameter for the third family of standard model quarks. Consider the mass of the top quark to be larger than the bottom quark mass: $m_t >> m_b$. You should find

$$
[\Pi_{++}(0) - \Pi_{33}(0)] = \frac{N_c}{16\pi^2}\frac{m_t^2}{4} + \mathcal{O}(\frac{m_b^2}{m_t^2}).
$$