Lectures on Composite Higgs

Adrián Carmona

Institute for Theoretical Physics,
ETH Zurich, 8093 Zurich, Switzerland

E-mail: carmona@itp.phys.ethz.ch

Contents

1 Non-Linear Realizations of a Symmetry

In this lecture, I will mostly follow Jose Santiago's notes, Pokorski [1], the review by Feruglio [2] as well as the original papers by CWZ [3] and CCWZ [4].

Due to several interesting features that we will review in Section 2, we will consider theories where the Higgs boson is identified with the pseudo Nambu-Goldstone bosons (pNGB) associated to the spontaneous breaking of some global symmetry G. One of the key ingredients in order to compute the corresponding low energy effective theory is the use of non-linear σ-models, that you have already encountered throughout this course. In the following we will review and introduce some useful concepts, most of which were first introduced in [3, 4].

Let us consider a real analytic manifold M, together with a Lie Group G acting on M

$$\begin{aligned} \varphi \;:\; G\times M &\longrightarrow M \\ (g,\Phi(x)) &\longmapsto T(g)\cdot\Phi(x) \end{aligned} \tag{1.1}$$

which we will assume hereinafter to be compact, connected and semi-simple. We will also assume that φ is analytical on its two arguments. The physical situation that we have in mind is that of a manifold of scalar fields $\Phi(x)$, with the origin describing the vacuum configuration Σ_0, whereas the Lie group G acting on these fields correspond to the symmetry group of the theory[1]. Let us call H to the continuous subgroup of G formed by all the elements of G leaving the origin invariant, i.e.,

$$H = \{h\in G \;:\; T(h)\cdot\Sigma_0 = \Sigma_0\} = \varphi(\,\cdot\,,\Sigma_0)^{-1}\left(\{\Sigma_0\}\right). \tag{1.2}$$

If we assume that $H \neq \varnothing$, we will be just dealing with the spontaneous symmetry breaking $G\to H$. Our goal is somehow to classify all possible non-linear realizations of this symmetry breaking. This is a very ambitious task that fortunately can be reduced to the study of some particular class of them, where H acts linearly on the manifold M. Those

[1]We will assume for the moment that this group is global, although we will change this later on.

for which this happens are said to be in the *standard form.* The first important step in this direction is provided by the so called Haag's theorem [5], which assures the existence of some particular type of coordinate transformation leading to equivalent physical theories.

Theorem 1 (Haag's Theorem) *If a theory is defined by a Lagrangian density*

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi) \tag{1.3}$$

depending on a set of scalar fields ϕ, and the following local transformation of fields is performed

$$\phi = F(\phi'), \tag{1.4}$$

then the transformed Lagrangian density:

$$\mathcal{L}'(\phi', \partial_\mu \phi') \tag{1.5}$$

defines a new theory with same $S-$matrix elements provided the transformation (1.4) has a Jacobian determinant equal to one at the origin.

The transformations fulfilling the conditions of the previous theorem will be called *allowed* ones. It turns out that it is always possible to choose coordinates on M (i.e., scalar field representations) such that the action of H on M is linear.

Theorem 2 *If H is the subgroup of G leaving the origin invariant, then it is always possible to choose coordinates on M so that*

$$T(h) \cdot \Phi(x) = D(h)\Phi(x), \qquad \forall h \in H, \tag{1.6}$$

where $D(h)$ is a linear representation of H.

As we already mentioned, this set of coordinates is said to be in *standard form.* Finally, it can be shown that any non-linear representation of the group G acting on M can be always brought to the standard form by means of an allowed coordinate transformation, so we just need to care about the study of the latter.

Theorem 3 *Any non-linear realization of G can be put into the standard form by an allowed coordinate transformation.*

Let be $\Phi_0(x)$ some field which transforms according to a linear representation of G, i.e.,

$$T(g) \cdot \Phi_0(x) = D(g)\Phi_0(x), \quad \forall g \in G. \tag{1.7}$$

In some neighborhood of the identity $g = e$, we can write an element of G as,

$$D(g) = \exp\left(-i\xi^{\hat{a}} X^{\hat{a}}\right) \exp\left(-iu^i Y^i\right) \tag{1.8}$$

where $X^{\hat{a}}$ and Y^i are the broken and unbroken generators, respectively, which we assume to be orthonormal with regard to the Cartan-Killing inner product, i.e., $\mathrm{Tr}(T^a T^b) = \delta_{ab}$ with $T^a, T^b \in \{X^{\hat{a}}, Y^i\}$. The scalar fields $\xi^{\hat{a}}(x)$ can be seen as the coordinates of the manifold of left cosets G/H (remember that $G/H = \{[l], l \in G\}$, where $[l] = \{lh,\ h \in H\}$) at each point of space-time. The decomposition (1.8) means that every element of the group $g \in G$ can be uniquely decomposed as a product $g = l(\xi)h$ where $h \in H$ and $l(\xi)$ is the representative member of the coset to which g belongs, i.e., $g \in [l]$ with some $l(\xi) \in G$. Let us define

$$\Phi_0(x) = \exp\left(-i\xi^{\hat{a}} X^{\hat{a}}\right) \Phi(x) \equiv U\Phi(x), \tag{1.9}$$

Let us now perform an arbitrary G transformation on the field Φ_0

$$\begin{aligned}\Phi_0'(x) &= \exp\left(-i\alpha_a T^a\right) \Phi_0(x) = \exp\left(-i\alpha_a T^a\right) \exp\left(-i\xi^{\hat{a}} X^{\hat{a}}\right) \Phi(x) \\ &= \exp\left(-i\alpha_a' T^a\right) \Phi(x) = \exp\left(-i\xi'^{\hat{a}}(\xi,\alpha) X^{\hat{a}}\right) \exp\left(-iu^i(\xi,\alpha) Y^i\right) \Phi(x), \end{aligned} \tag{1.10}$$

where in the last steps we have just used that

$$gl(\xi) = g'(g,\xi) = l(\xi')h, \tag{1.11}$$

where $h = h(g,\xi)$ and $\xi' = \xi'(g,\xi)$. We can then parametrize Φ_0 with the fields $\xi^{\hat{a}}(x)$ and $\Phi(x)$, transforming as follows

$$\xi^{\hat{a}}(x) \to \xi'^{\hat{a}}(x) = \xi'^{\hat{a}}(g, \xi(x)), \tag{1.12}$$

$$\Phi(x) \to \exp\left(-iu^i(x) Y^i\right) \Phi(x) = D(h(g,\xi(x)))\Phi(x). \tag{1.13}$$

This means that we can represent $\Phi_0(x)$ by the couple of fields $\{\xi(x), \Phi(x)\}$, where $\xi(x)$ transforms *non-linearly* under the global group G and Φ transforms *locally* under the unbroken subgroup H. What happens if the transformation is under the unbroken group H? In this case

$$\begin{aligned}\Phi_0'(x) &= \exp\left(-i\alpha_i Y^i\right) \exp\left(-i\xi^{\hat{a}} X^{\hat{a}}\right) \mathbf{1}\ \Phi(x) = \exp\left(-i\alpha_i Y^i\right) \exp\left(-i\xi^{\hat{a}} X^{\hat{a}}\right) \exp\left(i\alpha_i Y^i\right) \\ &\cdot\ \exp\left(-i\alpha_i Y^i\right) \Phi(x) = \exp\left(-i\xi^{\hat{a}} R_{\hat{a}\hat{b}} X^{\hat{b}}\right) \exp\left(-i\alpha_i Y^i\right) \Phi(x) \\ &= \exp\left(-i\xi'^{\hat{b}} X^{\hat{b}}\right) \exp\left(-i\alpha_i Y^i\right) \Phi(x), \end{aligned} \tag{1.14}$$

where $R_{\hat{a}\hat{b}}$ is the matrix representation of the linear transformation of the broken generators under the unbroken group,

$$\exp\left(-i\alpha_i Y^i\right) X^{\hat{a}} \exp\left(i\alpha_i Y^i\right) = R_{\hat{a}\hat{b}} X^{\hat{b}}. \tag{1.15}$$

We can thus see that under global transformations $h \in H$, both $\xi^{\hat{a}}(x)$ and $\Phi(x)$ transform linearly and globally

$$\xi^{\hat{a}}(x) \to R^T_{\hat{a}\hat{b}} \xi^{\hat{b}}(x), \tag{1.16}$$

$$\Phi(x) \to D(h)\Phi(x). \tag{1.17}$$

We would like to use this non-linear realization of the group to construct G-invariant Lagrangians. However, despite the fact that the symmetry is global, the non-linear realization involves the Goldstone fields $\xi^{\hat{a}}$ which transform locally. Thus derivatives have to be transformed into covariant derivatives. To this end, we will consider the following object[2]

$$\omega_\mu \equiv U^\dagger \partial_\mu U = e^{i\xi \cdot X} \partial_\mu e^{-i\xi \cdot X} = i d_\mu^{\hat{a}} X^{\hat{a}} + i E_\mu^i Y^i \equiv i d_\mu + i E_\mu. \tag{1.18}$$

If we compute

$$\begin{aligned} \partial_\mu \Phi_0 &= \partial_\mu \left[e^{-i\xi \cdot X} \Phi \right] = e^{-i\xi \cdot X} e^{i\xi \cdot X} \partial_\mu \left[e^{-i\xi \cdot X} \Phi \right] \\ &= e^{-i\xi \cdot X} \left[i d_\mu^{\hat{a}} X^{\hat{a}} \Phi + (\partial_\mu + i E_\mu^i Y^i) \Phi \right], \end{aligned} \tag{1.19}$$

and note that $\partial_\mu \Phi_0$ transforms under a global transformation g in the same was as Φ_0 does, one can readily conclude that

$$\left[i d_\mu^{\hat{a}} X^{\hat{a}} \Phi + (\partial_\mu + i E_\mu^i Y^i) \Phi \right] \to D(h(g, \xi(x))) \left[i d_\mu^{\hat{a}} X^{\hat{a}} \Phi + (\partial_\mu + i E_\mu^i Y^i) \Phi \right]. \tag{1.20}$$

As different representations of the unbroken group H are not mixed by $D(h(g, \xi(x)))$ it is clear that both terms inside the square brackets transform independently. Therefore,

$$\begin{aligned} d_\mu^{\hat{a}} X^{\hat{a}} \Phi &\to D(h(g, \xi(x))) d_\mu^{\hat{a}} X^{\hat{a}} \Phi = d_\mu'^{\hat{a}} X^{\hat{a}} \Phi' \\ &= D(h(g, \xi(x))) D(h(g, \xi(x)))^{-1} d_\mu'^{\hat{a}} X^{\hat{a}} D(h(g, \xi(x))) \Phi. \end{aligned} \tag{1.21}$$

This means in particular that,

$$\begin{aligned} d'_\mu &= D(h(g, \xi(x))) d_\mu D(h(g, \xi(x)))^{-1} = d_\mu^{\hat{b}} e^{-iu(g, \xi(x)) \cdot Y} X_{\hat{b}} e^{iu(g, \xi(x)) \cdot Y} \\ &= d_\mu^{\hat{b}} R_{\hat{b}\hat{a}}(g, \xi(x)) X^{\hat{a}}. \end{aligned} \tag{1.22}$$

Analogously,

$$\begin{aligned} \left(\partial_\mu + i E_\mu^i Y^i \right) \Phi &\to D(h(g, \xi(x))) \left(\partial_\mu + i E_\mu \right) \Phi = \left(\partial_\mu + i E'_\mu \right) \Phi' \\ &= D(h(g, \xi(x))) \left[\partial_\mu + i D(h(g, \xi(x)))^{-1} E'_\mu D(h(g, \xi(x))) \right. \\ &+ \left. D(h(g, \xi(x)))^{-1} \partial_\mu D(h(g, \xi(x))) \right] \Phi, \end{aligned} \tag{1.23}$$

and thus

$$\begin{aligned} E'_\mu &= D(h(g, \xi(x))) E_\mu D(h(g, \xi(x)))^{-1} + i \left[\partial_\mu D(h(g, \xi(x))) \right] D(h(g, \xi(x)))^{-1} \\ &= D(h(g, \xi(x))) E_\mu D(h(g, \xi(x)))^{-1} - i D(h(g, \xi(x))) \left[\partial_\mu D(h(g, \xi(x)))^{-1} \right]. \end{aligned} \tag{1.24}$$

Summarizing, under a global transformation $g \in G$,

$$\Phi \to D(h(g, \xi(x))) \Phi, \tag{1.25}$$

$$d_\mu \to D(h(g, \xi(x))) d_\mu D(h(g, \xi(x)))^{-1}, \qquad d_\mu^{\hat{a}} \to \left(R_{\hat{a}\hat{b}}(g, \xi(x)) \right)^T d_\mu^{\hat{b}}, \tag{1.26}$$

$$E_\mu \to D(h(g, \xi(x))) E_\mu D(h(g, \xi(x)))^{-1} - i D(h(g, \xi(x))) \partial_\mu D(h(g, \xi(x)))^{-1}. \tag{1.27}$$

[2]Technically, this is the so called Maurer-Cartan one form.

Moreover, the quantity $\mathcal{E}_\mu \equiv \partial_\mu + iE_\mu$ acts as an H-covariant derivative,

$$\mathcal{E}_\mu \Phi \to D(h(g,\xi(x)))\mathcal{E}_\mu \Phi. \tag{1.28}$$

With these building blocks we can construct in a very simple way G-invariant Lagrangians out of multiplets of the unbroken group. In particular, the Lagrangian describing the dynamics of the NGBs associated to G/H, at the level of two derivatives, is given by

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{4}\mathrm{Tr}\left(d_\mu d^\mu\right) = \frac{f_\pi^2}{4}\mathrm{Tr}\left[-iU^\dagger \partial_\mu U T^{\hat{a}}\right]\mathrm{Tr}\left[-iU^\dagger \partial^\mu U T^{\hat{a}}\right] \\ &= \frac{f_\pi^2}{4}(\partial_\mu \xi^{\hat{a}}(x))(\partial^\mu \xi^{\hat{a}}(x)) + \dots \end{aligned}\tag{1.29}$$

where f_π is a dimensionfull quantity that we need to have the correct mass dimensions [3].

What happens if some subgroup $H_0 \subset G$ is gauged? Essentially, the same formalism can be used, with the replacement of the usual derivative with the gauge covariant one,

$$\partial_\mu \to \partial_\mu + iA_\mu^{\dot{a}} T^{\dot{a}} = \partial_\mu + iA_\mu, \quad \text{where} \quad T^{\dot{a}} \in \mathrm{Alg}(H_0). \tag{1.30}$$

Defining

$$e^{i\xi\cdot X}\left[\partial_\mu + iA_\mu^{\dot{a}}T^{\dot{a}}\right]e^{-i\xi\cdot X} = i\bar{d}_\mu^{\hat{a}} X^{\hat{a}} + i\bar{E}_\mu^i Y^i = i\bar{d}_\mu + i\bar{E}_\mu, \tag{1.31}$$

where now $\bar{d}_\mu = \bar{d}_\mu(\xi, A)$ and $\bar{E}_\mu = \bar{E}_\mu(\xi, A)$.

$$\begin{aligned}(\partial_\mu + iA_\mu)\,\Phi_0 &= (\partial_\mu + iA_\mu)\,e^{-i\xi\cdot X}\Phi = e^{-i\xi\cdot X}\left(\partial_\mu + e^{i\xi\cdot X}\partial_\mu e^{-i\xi\cdot X} + ie^{i\xi\cdot X}A_\mu e^{-i\xi\cdot X}\right)\Phi \\ &= e^{-i\xi\cdot X}\left\{\partial_\mu + e^{i\xi\cdot X}\left[\partial_\mu + iA_\mu\right]e^{-i\xi\cdot X}\right\}\Phi \\ &= e^{-i\xi\cdot X}\left\{i\bar{d}_\mu + \left(\partial_\mu + i\bar{E}_\mu\right)\right\}\Phi. \end{aligned}\tag{1.32}$$

But since under a *local* G transformation, $(\partial_\mu + iA_\mu)\Phi_0$ transforms in the same way as Φ_0 does, that means that $[i\bar{d}_\mu + (\partial_\mu + i\bar{E}_\mu)]\Phi$ transforms as Φ does. Therefore, under *local* G transformations we have,

$$\Phi \to D(h(g(x),\xi(x)))\Phi, \tag{1.33}$$

$$\bar{d}_\mu \to D(h(g(x),\xi(x)))\bar{d}_\mu D(h(g(x),\xi(x)))^{-1}, \tag{1.34}$$

$$\bar{\mathcal{E}}_\mu \equiv (\partial_\mu + i\bar{E}_\mu)\Phi \to D(h(g(x),\xi(x)))\bar{\mathcal{E}}_\mu \Phi. \tag{1.35}$$

Now, the leading order Lagrangian for the gauge fields and the scalars $\xi^{\hat{a}}$ reads

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\bar{d}_\mu \bar{d}^\mu\right) - \frac{1}{4}F_{\mu\nu}^{\dot{a}}F^{\dot{a}\mu\nu} \tag{1.36}$$

where $F_{\mu\nu}^{\dot{a}}$ is the usual field-strength tensor. If we expand (1.36) in powers of $\xi^{\hat{a}}(x)$ and make the redefinition (see e.g. footnote 3) $\xi^{\hat{a}}(x) \to \sqrt{2}\xi^{\hat{a}}(x)/f_\pi$ we get

$$\mathcal{L} = \frac{1}{2}(D_\mu \xi^{\hat{a}}(x))^\dagger (D^\mu \xi^{\hat{a}}(x)) + \dots\ , \tag{1.37}$$

with canonically normalized kinetic terms.

[3] In particular, making the redefinition $\xi^{\hat{a}}(x) \to \sqrt{2}\xi^{\hat{a}}(x)/f_\pi$ we get canonically normalized kinetic terms.

2 Composite Higgs Models

In this section we will mostly follow Jose Santiago's notes as well as Roberto Contino's lectures [6].

2.1 General Picture

The main idea behind Composite Higgs models (CHM) is that the Higgs boson could emerge as a bound state of a strongly interacting sector, instead of being an elementary scalar. In this way, the quadratic sensitivity of the Higgs boson mass to the ultra-violet (UV) is saturated by new physics at some scale Λ before the new strong interaction featuring the Higgs as a bound state starts to be resolved. This provides a natural solution to what have been called the *hierarchy problem*. Moreover, if the Higgs is the pNGB associated to an enlarged global symmetry of the strong dynamics, the Higgs can be much lighter than the composite scale Λ. The main idea is sketched in Figure 1.

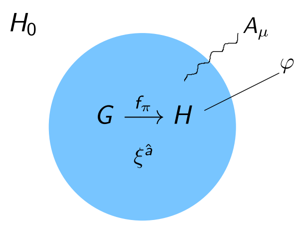
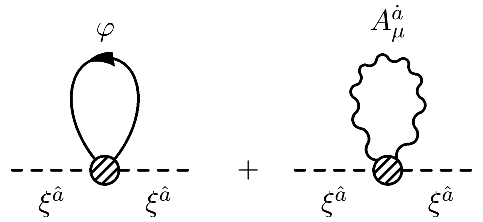


Figure 1. Sketch of the general picture in CHM. The spontaneous global symmetry breaking at the scale f_π, $G \to H$, delivers some NGB $\xi^{\hat{a}}(x)$. However, the gauging of some subgroup $H_0 \subset G$ generates a potential at the loop level for some of them that will be identified with the Higgs degrees of freedom.

We consider some global symmetry group G, which is spontaneously broken at the scale f_π to some subgroup H, delivering $n = \dim(G) - \dim(H)$ Nambu-Goldstone bosons (NGB) $\xi^{\hat{a}}(x)$. From these n NGB, $n_0 = \dim(H_0) - \dim(\mathcal{H})$ will be eaten to provide gauge boson longitudinal degrees of freedom after the gauging of the subgroup $H_0 \subset G$, where $\mathcal{H} \equiv H_1 \cap H_0$ is the unbroken gauge group. The remaining $n - n_0$ are thus pNGB which we will identify with the Higgs degrees of freedom. The interaction with the elementary sector, formed by gauge bosons and fermions transforming under H_0, will generate a potential for these degrees of freedom at the loop level. In particular

$m^2_{\xi^{\hat{a}}} =$



$. \qquad (2.1)$

This is also interesting as contrary to the SM case, where the Higgs potential is some ad-hoc term in the Lagrangian, in these models we have a dynamical explanation of the electroweak symmetry breaking (EWSB) and the Higgs mass.

2.2 Minimal Composite Higgs Models

We will consider now some explicit examples, focusing in particular in what is known by the Minimal Composite Higgs Model (MCHM).

2.2.1 Minimal Composite Higgs model (the real one)

Let us consider the minimal composite Higgs model. We want H_0 to include the electroweak group, so minimality requires it to *be* the electroweak (EW) group $H_0 = SU(2)_L \times U(1)_Y$. We also need to have a least four Goldstone bosons to be identify with the Higgs doublet. So, the minimal choice would be a group G with $8 = 4 + 4$ generators. The very first example which may come to our mind is $G = SU(3)$. Indeed, $SU(3)$ contains a $SU(2)$ and a additional $U(1)$ that we can try to identify with the EW group. We use the usual Gell-Mann matrices as the generators of $SU(3)$,

$$T^a = \frac{\lambda^a}{2}, \qquad a = 1, \ldots, 8, \tag{2.2}$$

with

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{2.3}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \tag{2.4}$$

They satisfy commutations relations, $[T^a, T^b] = i f_{abc} T^c$, with

$$f_{123} = 1, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2}, \tag{2.5}$$

$$f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}. \tag{2.6}$$

All the others (which are not related by total antisymmetry) are zero. In particular, we have

$$[T^i, T^j] = i\epsilon^{ijk} T^k, \quad [T^i, T^8] = 0, \quad i, j, k \in \{1, 2, 3\}, \tag{2.7}$$

so that T^a generate an $SU(2)$ subgroup and T^8 generates a $U(1)$ one, as anticipated. The coset space is panned by $T^{\hat{a}}$ with $\hat{a} = 4, 5, 6, 7$. Defining $T^+ \equiv T^4 - iT^5$ and $T^0 \equiv T^6 - iT^7$ and grouping them in a two dimensional vector,

$$T_\phi = \begin{pmatrix} T^+ \\ T^0 \end{pmatrix}, \tag{2.8}$$

and using the commutations relations, we get

$$[T^i, T_\phi] = -\frac{\sigma^i}{2} T_\phi, \qquad [T^8, T_\phi] = -\frac{\sqrt{3}}{2} T_\phi. \tag{2.9}$$

Therefore, using (1.15) we get

$$e^{-i\alpha_j T^j} T_\phi e^{i\alpha_k T^k} = T_\phi - i\alpha_j [T^j, T_\phi] + \ldots = (1 + i\alpha_j \frac{\sigma^j}{2}) T_\phi + \ldots = e^{i\alpha_j \frac{\sigma^j}{2}} T_\phi \tag{2.10}$$

and

$$\begin{pmatrix} \xi^+(x) \\ \xi^0(x) \end{pmatrix}^* \to \left(e^{i\alpha_j \frac{\sigma^j}{2}} \right)^T \begin{pmatrix} \xi^+(x) \\ \xi^0(x) \end{pmatrix}^* \Rightarrow \begin{pmatrix} \xi^+(x) \\ \xi^0(x) \end{pmatrix} \to e^{-i\alpha_j \frac{\sigma^j}{2}} \begin{pmatrix} \xi^+(x) \\ \xi^0(x) \end{pmatrix}. \tag{2.11}$$

Thus one can see that

$$\begin{pmatrix} \xi^+(x) \\ \xi^0(x) \end{pmatrix} \tag{2.12}$$

have the correct quantum numbers to be identified with the SM Higgs. Let us define

$$\begin{aligned} U &\equiv \exp\left(-i \frac{2}{f_\pi} \xi^{\hat{a}}(x) T^{\hat{a}} \right) \qquad (2.13) \\ &= \begin{pmatrix} \frac{(\xi^6)^2+(\xi^7)^2+\left((\xi^4)^2+(\xi^5)^2\right)\cos\left(\frac{\xi}{f_\pi}\right)}{\xi^2} & \frac{(\xi^4-i\xi^5)(\xi^6+i\xi^7)\left(\cos\left(\frac{\xi}{f_\pi}\right)-1\right)}{\xi^2} & \frac{(-i\xi^4-\xi^5)\sin\left(\frac{\xi}{f_\pi}\right)}{\xi} \\ \frac{(\xi^4+i\xi^5)(\xi^6-i\xi^7)\left(\cos\left(\frac{\xi}{f_\pi}\right)-1\right)}{\xi^2} & \frac{(\xi^4)^2+(\xi^5)^2+\left((\xi^6)^2+(\xi^7)^2\right)\cos\left(\frac{\xi}{f_\pi}\right)}{\xi^2} & \frac{(-i\xi^6-\xi^7)\sin\left(\frac{\xi}{f_\pi}\right)}{\xi} \\ \frac{(\xi^5-i\xi^4)\sin\left(\frac{\xi}{f_\pi}\right)}{\xi} & \frac{(\xi^7-i\xi^6)\sin\left(\frac{\xi}{f_\pi}\right)}{\xi} & \cos(\frac{\xi}{f_\pi}) \end{pmatrix}, \end{aligned}$$

where $\xi \equiv \left(\sum_{\hat{a}} (\xi^{\hat{a}})^2\right)^{1/2}$. For the sake of simplicity we can go to the unitary gauge, where three of the NGBs are eaten by the gauge bosons. In such a gauge, it is always possible to take the physical Higgs h to be aligned with ξ^6, i.e. $h \equiv \xi^6$, without loss of generality, obtaining

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(h/f_\pi) & -i\sin(h/f_\pi) \\ 0 & -i\sin(h/f_\pi) & \cos(h/f_\pi) \end{pmatrix}. \tag{2.14}$$

Therefore,

$$\mathcal{L}_\xi = \frac{f_\pi^2}{4} \left(\bar{d}^{\hat{a}}_\mu \bar{d}^{\hat{a}\mu} \right) = \frac{f_\pi^2}{2} \mathrm{Tr}\left[-iU^\dagger D_\mu U T^{\hat{a}} \right] \mathrm{Tr}\left[-iU^\dagger D^\mu U T^{\hat{a}} \right], \tag{2.15}$$

and

$$\mathcal{L} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{f_\pi^2}{2} \mathrm{Tr}\left[-iU^\dagger D_\mu U T^{\hat{a}} \right] \mathrm{Tr}\left[-iU^\dagger D^\mu U T^{\hat{a}} \right] \tag{2.16}$$

where in (2.15) we have used that $\mathrm{Tr}\left(T^a \cdot T^b\right) = \delta^{ab}/2$ and the covariant derivative read

$$\begin{aligned} D_\mu &= \partial_\mu - igW^j_\mu T^j - ig' B_\mu Y \\ &= \partial_\mu - igW^\pm_\mu T^\pm - i\frac{g}{c_W}(T^3 - s_W^2 Q) Z_\mu - igs_W A_\mu Q \end{aligned} \tag{2.17}$$

where we have defined

$$A_\mu \equiv s_W W^3_\mu + c_W B_\mu, \qquad Z_\mu \equiv c_W W^3_\mu - s_W B_\mu, \tag{2.18}$$

$$c_W \equiv \frac{g}{\sqrt{g^2+g'^2}}, \qquad s_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}, \tag{2.19}$$

$$W^\pm_\mu \equiv \frac{W^1 \mp iW^2}{\sqrt{2}}, \qquad T^\pm \equiv \frac{T^1 \pm iT^2}{\sqrt{2}}, \tag{2.20}$$

and

$$Q \equiv T^3 + Y, \quad Y \equiv \sqrt{\frac{1}{3}} T^8. \tag{2.21}$$

This leads in particular to

$$\bar{d}^4_\mu = -\frac{1}{2} ig(W^-_\mu - W^+_\mu)\sin\left(\frac{h}{f_\pi}\right), \quad \bar{d}^5_\mu = -\frac{1}{2} g(W^-_\mu + W^+_\mu)\sin\left(\frac{h}{f_\pi}\right), \tag{2.22}$$

$$\bar{d}^6_\mu = -\sqrt{2}\partial_\mu h, \quad \bar{d}^7_\mu = \frac{1}{2\sqrt{2}}\frac{g}{c_W} Z_\mu \sin\left(\frac{2h}{f_\pi}\right), \tag{2.23}$$

and

$$\mathcal{L}_\xi = \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) + \frac{g^2}{4} f_\pi^2 \sin^2\left(\frac{h}{f_\pi}\right) W^+_\mu W^{-\mu} + \frac{g^2}{32 c_W^2} f_\pi^2 \sin^2\left(2\frac{h}{f_\pi}\right) Z_\mu Z^\mu. \tag{2.24}$$

Looking at the above expression one could notice already that we have encounter some phenomenological problem. In order to see it more explicitly, let us assume that that the Higgs boson gets a vev $\langle h \rangle = \tilde{v}$, which will give masses to the EW bosons $W^\pm$ and Z,

$$m_W^2 = \frac{g^2}{4} f_\pi^2 \sin^2\left(\frac{\tilde{v}}{f_\pi}\right) = \frac{g^2}{4}\tilde{v}^2\left(1 - \frac{1}{3}\frac{\tilde{v}^2}{f_\pi^2} + \mathcal{O}(\tilde{v}^4/f_\pi^4)\right), \tag{2.25}$$

$$m_Z^2 = \frac{g^2}{16 c_W^2} f_\pi^2 \sin^2\left(2\frac{\tilde{v}}{f_\pi}\right) = \frac{g^2}{4 c_W^2}\tilde{v}^2\left(1 - \frac{4}{3}\frac{\tilde{v}^2}{f_\pi^2} + \mathcal{O}(\tilde{v}^4/f_\pi^4)\right). \tag{2.26}$$

If we compute the ρ parameter

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1 + \frac{\tilde{v}^2}{f_\pi^2} + \mathcal{O}(\tilde{v}^4/f_\pi^4), \tag{2.27}$$

we can see that in order to be in agreement with EW precision data, we need to increase f_π significantly in order to make $\tilde{v}^2/f_\pi^2$ small enough. Taking into account that $v \equiv v_{\rm SM} = f_\pi \sin(\tilde{v}/f_\pi) \sim v$, this leads to $f_\pi \sim \mathcal{O}(10~\text{TeV})$, putting any possible experimental probe of this idea beyond current experiments. Moreover, as we will see later, the tuning of these models scales with $f_\pi/\tilde{v}$, making this particular model not too compelling. One possibility of curing this problem is to incorporate custodial symmetry and we will consider this case in the following.

2.2.2 Minimal Composite Higgs model (the custodial one)

As mentioned, we can protect the ρ parameter from new physics corrections with the help of the custodial symmetry. To do so, we just need to make sure that $SU(2)_L \times SU(2)_R \subset G$. It can be seen that the minimal choice for G would be then $SO(5)$. However, at the end of the day, if one wants also to add the interaction with the elementary fermions of the SM (as we do), in order to reproduce the correct hypercharges it is required to add an additional $U(1)_X$ under which the Higgs is not charged. Thus, we will consider $G = SO(5) \times U(1)_X$ and $H = SO(4) \times U(1)_X$. Let us assume for concreteness the following basis for the $SO(5)$ generators, $\{T^\alpha,\ \alpha = 1, \dots, 10\} = \{T^a_L, T^b_R, T^{\hat{a}}_C,\ a, b = 1, 2, 3,\ \hat{a} = 1, \dots, 4\}$, in the fundamental representation

$$\begin{aligned}
T^a_{L,ij} &= -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}\left(\delta^b_i\delta^c_j - \delta^b_j\delta^c_i\right) + \left(\delta^a_i\delta^4_j - \delta^a_j\delta^4_i\right)\right], \qquad a = 1, 2, 3,\\
T^a_{R,ij} &= -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}\left(\delta^b_i\delta^c_j - \delta^b_j\delta^c_i\right) - \left(\delta^a_i\delta^4_j - \delta^a_j\delta^4_i\right)\right], \qquad a = 1, 2, 3,\\
T^{\hat{a}}_{C,ij} &= -\frac{i}{\sqrt{2}}\left[\delta^{\hat{a}}_i\delta^5_j - \delta^{\hat{a}}_j\delta^5_i\right], \qquad \hat{a} = 1, 2, 3, 4,
\end{aligned} \tag{2.28}$$

that have been chosen fulfilling $\mathrm{Tr}\left(T^\alpha \cdot T^\beta\right) = \delta^{\alpha\beta}$. They satisfy the following commutation relations

$$\left[T^a_L, T^b_L\right] = i\epsilon^{abc}T^c_L, \quad \left[T^a_R, T^b_R\right] = i\epsilon^{abc}T^c_R, \quad \left[T^a_L, T^b_R\right] = 0, \tag{2.29}$$

$$\left[T^a_C, T^b_C\right] = \frac{i}{2}\epsilon^{abc}(T^c_L + T^c_R), \quad [T^a_C, T^4_C] = \frac{i}{2}(T^a_L - T^a_R), \tag{2.30}$$

$$\left[T^a_{L,R}, T^b_C\right] = \frac{i}{2}(\epsilon^{abc}T^c_C \pm \delta^{ab}T^4_C), \quad [T^a_{L,R}, T^4_C] = \mp\frac{i}{2}T^a_C. \tag{2.31}$$

From the previous commutation relations it can be seen that $SU(2)_L \times SU(2) \cong SO(4) \subset SO(5)$ and that the generators in the coset space (and the corresponding Goldstone bosons) transform as a $(\mathbf{2}, \mathbf{2})$ of $SU(2)_L \times SU(2)_R$ (or equivalently, a $\mathbf{4}$ of $SO(4)$). Indeed, if we define

$$T_\phi = \begin{pmatrix} T^2_C + iT^1_C \\ T^4_C - iT^3_C \end{pmatrix} \tag{2.32}$$

we obtain

$$[T^a_L, T_\phi] = -\frac{1}{2}\sigma^a T_\phi, \quad [T^3_R, T_\phi] = -\frac{1}{2}T_\phi, \tag{2.33}$$

and thus

$$\begin{pmatrix} \xi^2 + i\xi^1 \\ \xi^4 - i\xi^3 \end{pmatrix} \to e^{-i\alpha_a\frac{\sigma^a}{2}} \begin{pmatrix} \xi^2 + i\xi^1 \\ \xi^4 - i\xi^3 \end{pmatrix}, \tag{2.34}$$

under a global $SU(2)_L$ rotation, having therefore the correct quantum numbers to be identified with the Higgs doublet. Looking at the generators (2.29) it is clear that one can take the following vacuum

$$\Sigma_0^T \equiv (0, 0, 0, 0, 1), \tag{2.35}$$

which satisfies

$$T_L^a \cdot \Sigma_0 = 0, \qquad T_R^a \cdot \Sigma_0 = 0, \qquad a = 1,2,3, \tag{2.36}$$

and

$$\frac{1}{2}\mathrm{Tr}(T_C^{\hat{a}} \cdot T_C^{\hat{b}}) = \Sigma_0^T \cdot T_C^{\hat{a}} \cdot T_C^{\hat{b}} \cdot \Sigma_0 = \frac{1}{2}\delta^{\hat{a}\hat{b}} \tag{2.37}$$

Therefore, if we define [4]

$$\Sigma \equiv U \cdot \Sigma_0 = \exp\left(i\sqrt{2}T_C^{\hat{a}}\xi^{\hat{a}}(x)/f_\pi\right) \cdot \Sigma_0 \tag{2.38}$$

we can write

$$D_\mu \Sigma = iU\bar{d}_\mu^{\hat{a}} T_C^{\hat{a}} \Sigma_0 \tag{2.39}$$

and thus

$$\begin{aligned} \frac{f_\pi^2}{2}\left(D_\mu\Sigma\right)^\dagger \cdot \left(D^\mu\Sigma\right) &= \frac{f_\pi^2}{2}\bar{d}_\mu^{\hat{a}}\bar{d}^{\hat{b}\mu}\left[-i\Sigma_0^T T_C^{\hat{a}} U^\dagger\right] \cdot \left[iUT_C^{\hat{b}}\Sigma_0\right] \\ &= \frac{f_\pi^2}{2}\bar{d}_\mu^{\hat{a}}\bar{d}^{\hat{b}\mu}\left(\Sigma_0^T \cdot T_C^{\hat{a}} \cdot T_C^{\hat{b}} \cdot \Sigma_0\right) = \frac{f_\pi^2}{4}\bar{d}_\mu^{\hat{a}}\bar{d}^{\hat{a}\mu}. \end{aligned} \tag{2.40}$$

In particular,

$$\Sigma^T = \frac{\sin(\hat{\xi}/f_\pi)}{\hat{\xi}}\left(\xi^1, \xi^2, \xi^3, \xi^4, \hat{\xi}\cot(\hat{\xi}/f_\pi)\right), \qquad \hat{\xi} \equiv \left(\sum_{\hat{a}}\left(\xi^{\hat{a}}\right)^2\right)^{1/2}. \tag{2.41}$$

Without loss of generality, we can assume that, in the unitary gauge,

$$\Sigma^T = (0, 0, 0, \sin(h/f_\pi), \cos(h/f_\pi)), \tag{2.42}$$

where $h(x) \equiv \xi^4(x)$.

Gauge Bosons As shown in (2.40), we can write the low energy effective Lagrangian as follows

$$\mathcal{L} = \frac{f_\pi^2}{2}\left(D_\mu\Sigma\right)^\dagger\left(D^\mu\Sigma\right) - \frac{1}{4}W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{2.43}$$

with

$$\begin{aligned} D_\mu &= \partial_\mu - igW_\mu^a T_L^a - ig' B_\mu Y \\ &= \partial_\mu - igW_\mu^\pm T_L^\pm - i\frac{g}{c_W}(T_L^3 - s_W^2 Q)Z_\mu - igs_W A_\mu Q \end{aligned} \tag{2.44}$$

and[5]

$$Y \equiv T_R^3 + Q_X, \qquad Q \equiv T_L^3 + T_R^3 + Q_X, \qquad T_L^\pm \equiv \frac{1}{\sqrt{2}}\left(T_L^1 \pm iT_L^2\right), \tag{2.45}$$

[4] For convenience, we are rescaling the pNGB with an additional minus sign, compared to previous sections, i.e., $\xi^{\hat{a}} \to -\sqrt{2}\xi^{\hat{a}}/f_\pi$.

[5] Remember that the Goldstones are not charged under $U(1)_X$ and thus $Q_X = 0$.

leading to

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) + \frac{g^2}{4} f_\pi^2 \sin^2\left(\frac{h}{f_\pi}\right) W_\mu^+ W^{-\mu} + \frac{g^2}{8c_W^2} f_\pi^2 \sin^2\left(\frac{h}{f_\pi}\right) Z_\mu Z^\mu \\ &- \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \end{aligned} \tag{2.46}$$

Assuming that, as we will see, the Higgs takes a vev $\langle h \rangle = \tilde{v} \neq 0$ and expanding around this value $h \to \tilde{v} + h$ we get

$$\begin{aligned} f_\pi^2 \sin^2\left(\frac{h}{f_\pi}\right) \to& f_\pi^2 \left[\sin^2\left(\frac{\tilde{v}}{f_\pi}\right) + 2\sin\left(\frac{\tilde{v}}{f_\pi}\right)\cos\left(\frac{\tilde{v}}{f_\pi}\right)\left(\frac{h}{f_\pi}\right) \right. \\ &\left. + \left(1 - 2\sin^2\left(\frac{\tilde{v}}{f_\pi}\right)\right)\left(\frac{h}{f_\pi}\right)^2 + \dots \right] \\ =& \; v^2 + 2v\sqrt{1 - v^2/f_\pi^2}\, h + \left(1 - 2v^2/f_\pi^2\right) h^2 + \dots \;, \end{aligned} \tag{2.47}$$

where we have used that, analogously to the $SU(3)$ case, in order to get the physical W mass,

$$v = f_\pi \sin\left(\frac{\tilde{v}}{f_\pi}\right). \tag{2.48}$$

In particular, that means that the SM couplings to the gauge bosons $V = W, Z$ are modified as follows

$$g_{VVh} = g_{VVh}^{\mathrm{SM}} \sqrt{1 - v^2/f_\pi^2}, \qquad g_{VVhh}^{\mathrm{SM}} = g_{VVhh}^{\mathrm{SM}} (1 - 2v^2/f_\pi^2). \tag{2.49}$$

On the other hand, if we compute again the ρ parameter, contrary to the $SU(3)$ case, we get

$$\rho = \frac{m_W^2}{m_Z^2 c_W^2} = 1, \tag{2.50}$$

with no new physics corrections modifying this value, which is a direct consequence of the custodial setup of the model.

We want now to obtain some information about the gauge boson contribution to the Coleman-Weinberg (CW) potential. In order to do so, we write the most general $SO(5) \times U(1)_X$ invariant Lagrangian build out of the Goldstone bosons and the external (elementary) gauge fields, adding some *spurions* gauge fields, so that the external gauge fields form a complete adjoint representation of $SO(5) \times U(1)_X$. At the quadratic level and in momentum space, the relevant Lagrangian reads

$$\mathcal{L} = \frac{1}{2} \left(\mathcal{P}_T\right)^{\mu\nu} \left[\Pi_0^X(p^2) X_\mu X^\mu + \Pi_0(p^2) \mathrm{Tr}\left(A_\mu \cdot A_\nu\right) + \Pi_1(p^2) \Sigma^T \cdot A_\mu \cdot A_\nu \cdot \Sigma\right], \tag{2.51}$$

where X_μ is the gauge boson associated to $U(1)_X$ and we have defined

$$A_\mu \equiv A_\mu^\alpha T^\alpha = W_{L\mu}^a T_L^a + W_{R\mu}^a T_R^a + A_\mu^{\hat{a}} T_C^{\hat{a}} \tag{2.52}$$

and

$$(\mathcal{P}_T)_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \tag{2.53}$$

is the transverse projector so that terms with two gauge fields are the (quadratic) transverse part of $F_{\mu\nu}$, which transforms as $F_{\mu\nu} \to \Omega F_{\mu\nu}\Omega^{-1}$. Also we have $\Sigma \to \Omega\Sigma$. Since we want to derive only the Higgs potential and not its derivative interactions, the field Σ has been treated as a classical background, with vanishing momentum. The form factors $\Pi_0^X, \Pi_{0,1}$ encode the dynamics of the strong sector, including the effect of the fluctuations around the background Σ. If we switch off the unphysical gauge fields for a moment, keeping only those of $SU(2)_L \times U(1)_Y$, we obtain

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(\mathcal{P}_T)^{\mu\nu}\Bigg[&\left(\Pi_0(p^2) + \frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2)\right)W^a_\mu W^a_\nu - 2s_X\frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2)W^3_\mu B_\nu \\ &\left(c_X^2\Pi_0^X(p^2) + s_X^2\left(\Pi_0(p^2) + \frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2)\right)\right)B_\mu B^\mu\Bigg],\end{aligned} \tag{2.54}$$

where $W_\mu = W_{L\mu}$ and the hypercharge gauge boson B_μ is embedded as follows

$$B_\mu = s_X W^3_{R\mu} + c_X X_\mu, \tag{2.55}$$

with

$$s_X \equiv \frac{g_X}{\sqrt{g^2+g_X^2}} = \frac{g'}{g}, \qquad c_X \equiv \frac{g}{\sqrt{g^2+g_X^2}} = \sqrt{1-\frac{g'^2}{g^2}}. \tag{2.56}$$

and thus $W^3_{R\mu}\Big|_{\text{phys}} = s_X B_\mu$, $X_\mu|_{\text{phys}} = c_X B_\mu$. If we call

$$\Pi_{WW} = \Pi_0(p^2) + \frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2), \tag{2.57}$$

$$\Pi_{BB} = c_X^2\Pi_0^X(p^2) + s_X^2\left(\Pi_0(p^2) + \frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2)\right), \tag{2.58}$$

$$\Pi_{W_3B} = = -s_X\frac{\sin^2(h/f_\pi)}{4}\Pi_1(p^2), \tag{2.59}$$

we can write

$$\mathcal{L} = \frac{1}{2}(\mathcal{P}_T)^{\mu\nu}\left[2\Pi_{WW}W^+_\mu W^-_\nu + \Pi_{WW}W^3_\mu W^3_\mu + \Pi_{BB}B_\mu B_\nu + 2\Pi_{WB}W^3_\mu B_\nu\right]. \tag{2.60}$$

Moreover, after EWSB, the form factor Π_{WB} is related to the S-parameter [7]:

$$\Delta S = -\frac{16\pi}{gg'}\Pi'_{WB}(0) \approx \frac{16\pi}{g^2}\frac{\sin^2(\tilde{v}/f_\pi)}{4}\Pi'_1(0), \tag{2.61}$$

where $\Delta S = S - S_{\text{SM}}$ and $'$ stands for a derivative with respect to p^2. This is the more important constraint coming from EW precision data and typically implies that $v^2/f_\pi^2 \equiv \sin^2(\tilde{v}/f_\pi) < 1$. If we expand the above two-point functions in powers of p^2 and take

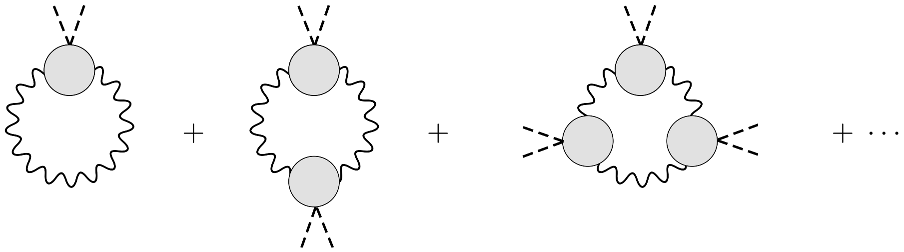


Figure 2. One-loop contribution of the SM gauge fields to the Higgs potential. A grey blob represents the strong dynamics encoded by the form factor Π_1. Figure taken from [6].

the leading terms, obtained by evaluating them at zero-momentum, we should recover the results of (2.46). This leads in particular to

$$\Pi_0(0) = 0 = \Pi_0^X(0), \qquad \Pi_1(0) = g^2 f_\pi^2, \tag{2.62}$$

where we have used that

$$\Pi_{WW}(0) = \Pi_0(0) + \frac{\sin^2(h/f_\pi)}{4}\Pi_1(0) = \frac{g^2}{4} f_\pi^2 \sin^2(h/f_\pi), \tag{2.63}$$

and

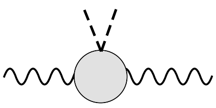
$$\begin{pmatrix} 0 & 0 \\ 0 & \frac{g^2}{4c_W^2} f_\pi^2 \sin^2(h/f_\pi) \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} \Pi_{BB}(0) & \Pi_{WB}(0) \\ \Pi_{WB}(0) & \Pi_{WW}(0) \end{pmatrix} \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} =$$
$$\begin{pmatrix} c_W^2 \Pi_{BB}(0) + s_W^2 \Pi_{WW}(0) + 2c_W s_W \Pi_{WB}(0) & c_W s_W \left[\Pi_{WW}(0) - \Pi_{BB}(0)\right] + (c_W^2 - s_W^2)\Pi_{WB}(0) \\ c_W s_W \left[\Pi_{WW}(0) - \Pi_{BB}(0)\right] + (c_W^2 - s_W^2)\Pi_{WB}(0) & s_W^2 \Pi_{BB}(0) + c_W^2 \Pi_{WW}(0) - 2c_W s_W \Pi_{WB}(0) \end{pmatrix}.$$

Let now study the gauge contribution to the one-loop Coleman-Weinberg potential. For the sake of simplicity, we will neglect hereinafter the contribution coming from the hypercharge gauge boson B_μ. After adding the following gauge-fixing term

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\zeta}\left(\partial^\mu W_\mu^a\right)^2, \tag{2.64}$$

we obtain the following expressions for the gauge boson propagator and its effective interaction with the Higgs [6]

$$G_{\mu\nu} = \frac{i}{\Pi_0(p^2)}(\mathcal{P}_T)_{\mu\nu} - \zeta\frac{i}{p^2}(\mathcal{P}_L)_{\mu\nu},$$



$$i\Gamma_{\mu\nu} = \frac{i\Pi_1(p^2)}{4}\sin^2(h/f_\pi)(\mathcal{P}_T)_{\mu\nu},$$

where $(\mathcal{P}_L)_{\mu\nu} = p^\mu p^\nu/p^2$ is the longitudinal projector. The one-loop Coleman-Weinberg potential can be computed resumming the infinite series of diagrams shown in Figure 2. One gets, integrating in Euclidean space,

$$V_g(h) = \frac{9}{2}\int_0^\infty \frac{\mathrm{d}^4 p_E}{(2\pi)^4}\log\left(1 + \frac{1}{4}\frac{\Pi_1(-p_E^2)}{\Pi_0(-p_E^2)}\sin^2(h/f_\pi)\right), \tag{2.65}$$

where the factor 9 arises from summing over the three polarizations and the three $SU(2)$ gauge fields. We can obtain some more information going to the so called large-N limit, see [8–10]. One assumes a $SU(N)$ gauge theory and that it is a confining theory for large N. Then it is possible to write the n-point Green functions of quark bilinears, in the large N-limit, as an infinite sum over stable intermediate meson resonances created out of the vacuum. In particular, that means that

$$\langle J_a^\mu J_a^\nu \rangle \equiv \langle 0|T\{J_a^\mu J_a^\nu\}|0\rangle = (\mathcal{P}_T)^{\mu\nu}\,\Pi_a(p^2) = \left(p^2\eta^{\mu\nu} - p^\mu p^\nu\right) g^2 \sum_{n=1}^{\infty} \frac{f_{\rho_n}^2}{p^2 - m_{\rho_n}}, \tag{2.66}$$

$$\begin{aligned}\langle J_{\hat{a}}^\mu J_{\hat{a}}^\nu \rangle &\equiv \langle 0|T\{J_{\hat{a}}^\mu J_{\hat{a}}^\nu\}|0\rangle = (\mathcal{P}_T)^{\mu\nu}\,\Pi_{\hat{a}}(p^2) \\ &= \left(p^2\eta^{\mu\nu} - p^\mu p^\nu\right) g^2 \left[\sum_{n=1}^{\infty} \frac{f_{a_n}^2}{p^2 - m_{a_n}} + \frac{1}{p^2}\frac{f_\pi^2}{2}\right],\end{aligned} \tag{2.67}$$

where ρ_n and a_n, $n \in \mathbb{N}$, are the tower of vector resonances associated to the broken $T^{\hat{a}}$ and unbroken T^a generators, respectively, and the last term in (2.67) correspond to the corresponding massless NGBs. If we expand the Lagrangian (2.51) around the $SO(4)$-preserving vacuum Σ_0 and use that

$$\Sigma_0^T \cdot T_C^{\hat{a}} \cdot T_C^{\hat{b}} \cdot \Sigma = \frac{1}{2}\mathrm{Tr}(T_C^{\hat{a}} \cdot T_C^{\hat{b}}) = \frac{1}{2}\delta^{\hat{a}\hat{b}}, \qquad T_L^a \cdot \Sigma_0 = T_R^a \cdot \Sigma_0 = 0, \tag{2.68}$$

we obtain

$$\Pi_a(p^2) = \Pi_0(p^2), \qquad \Pi_{\hat{a}}(p^2) = \Pi_0 + \frac{1}{2}\Pi_1(p^2). \tag{2.69}$$

That means in particular that

$$\Pi_0(p^2) = p^2 g^2 \sum_{n=1}^{\infty} \frac{f_{\rho_n}^2}{p^2 - m_{\rho_n}}, \tag{2.70}$$

$$\Pi_1(p^2) = g^2 f_\pi^2 + 2p^2 g^2 \left[\sum_{n=1}^{\infty} \frac{f_{a_n}^2}{p^2 - m_{a_n}} - \sum_{n=1}^{\infty} \frac{f_{\rho_n}^2}{p^2 - m_{\rho_n}}\right], \tag{2.71}$$

Note that the above form factors will generically lead to modifications of the gauge couplings g and g' introduced in (2.44) after canonically normalize the gauge kinetic terms. In particular, after adding also the bare kinetic terms of (2.43) one gets

$$\frac{1}{g_{\text{phys}}^2} = -\frac{\Pi_0'(0)}{g^2} + \frac{1}{g^2} \Rightarrow g_{\text{phys}}^2 = g^2\left(1 + \sum_{n=1}^{\infty} \frac{g^2}{g_{\rho_n}^2}\right)^{-1}, \qquad g_{\rho_n} \equiv m_{\rho_n}/f_{\rho_n}. \tag{2.72}$$

At energies much above the scale of symmetry breaking, the $SO(5)$ invariance is restored, and the difference of two-point functions along broken and unbroken directions is expected to vanish, i.e.,

$$\lim_{p_E^2 \to \infty} g^{-2}\Pi_1(-p_E^2) = f_\pi^2 + 2\sum_{n=1}^{\infty} f_{a_n}^2 - 2\sum_{n=1}^{\infty} f_{\rho_n}^2 = 0. \tag{2.73}$$

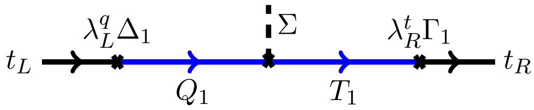


Figure 3. Schematic description of the mechanism giving rise to the fermion masses in the framework of partial compositeness.

The above condition, which relates the spectra of "vector" and "axial" currents, is known as the first Weinberg sum rule [11]. In particular, this is also telling us that for large Euclidean momenta $\Pi_1(-p_E^2)$ goes at least like p_E^{-2}. This means that the gauge contribution to the Higgs potential,

$$V_g(h) = \frac{9}{32\pi^2} \int_0^\infty \mathrm{d}p_E^2 p_E^2 \log\left(1 + \frac{1}{4}\frac{\Pi_1(-p_E^2)}{\Pi_0(-p_E^2)} \sin^2(h/f_\pi)\right), \tag{2.74}$$

is at most logarithmically divergent, as $\Pi_0'(0) \neq 0 \Rightarrow \Pi_0(-p_E^2) \propto p_E^2$ for large Euclidean momenta. In some specific UV completions of these scenarios, like holographic composite Higgs models [12, 13] or their discretized n-site versions [14, 15], the above contribution to the Higgs potential is actually finite. In this case, one would have

$$\lim_{p_E^2 \to \infty} g^{-2} p_E^2 \Pi_1(-p_E^2) = 2\sum_{n=1}^{\infty} f_{a_n}^2 m_{a_n}^2 - 2\sum_{n=1}^{\infty} f_{\rho_n}^2 m_{\rho_n}^2 = 0, \tag{2.75}$$

condition which is known as the second Weinberg sum rule.

Fermions Fermions are added in the framework of composite Higgs models through linear mixings to composite operators, i.e.,

$$\mathcal{L}_{\text{mix}} = \lambda_L^q \bar{q}_L \mathcal{O}_L^q + \lambda_R^t \bar{t}_R \mathcal{O}_R^t + \text{h.c.} \quad \langle 0|\mathcal{O}_L^q|Q_n\rangle = \Delta_n \quad \langle 0|\mathcal{O}_R^t|T_n\rangle = \Gamma_n, \tag{2.76}$$

which induces the low energy effective Lagrangian

$$\mathcal{L}_{\text{mix}} = \lambda_L^q \Delta_1 \bar{q}_L Q_{1R} + \lambda_R^t \Gamma_1 \bar{t}_R T_{1R} + \text{h.c.} + \dots \,. \tag{2.77}$$

The SM states will be a mixture of elementary and composite states, with masses after EWSB given by

$$m_t \sim \frac{v}{\sqrt{2}} \frac{\lambda_L^q \Delta_1}{m_{Q_1}} \frac{\lambda_R^t \Gamma_1}{m_{T_1}} \frac{Y}{f_\pi}. \tag{2.78}$$

This mechanism is thus known by the name of *partial compositeness.*

Similarly to the gauge boson case, the linear interactions of the elementary fermions to the composite sector explicitly break the Goldstone symmetry and generate thus a contribution to the Coleman-Weinberg potential. The breaking of the Goldstone symmetry and thus the size of the contribution to the loop potential will depend on the size of the

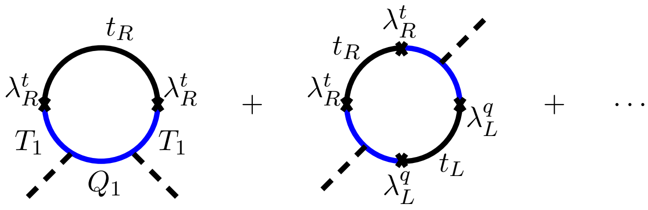


Figure 4. Schematic representation of the top contribution to the one-loop Coleman-Weinberg potential.

linear mixings in equation (2.77). As the fermion masses are also controlled by the same linear mixings, as depicted in Figure 3, one expects the top quark – the heaviest elementary particle in the spectrum – to give the most important contribution. Then, neglecting contribution of leptons and light quarks as a first approximation, one would have sum up an infinite number of loop diagrams as the ones shown in Figure 4. However, one could still get very useful information performing an spurion analysis, analogously to what we did in the gauge boson case.

We will assume for concreteness that the composite operators coupled to the elementary fermions transform in fundamental representations of $SO(5)$. For simplicity, let us introduce just one fermion excitation of such operators, ψ, transforming also as a **5** of $SO(5)$. Similarly to what we did for the case of scalars in Section 1, one can decompose this fermionic multiplet of $SO(5)$ in its $SO(4)$ components with the help of the U matrix defined in (2.38),

$$\psi = U \begin{pmatrix} Q \\ T \end{pmatrix}, \tag{2.79}$$

where $Q \sim (\mathbf{2}, \mathbf{2})$ and $T \sim (\mathbf{1}, \mathbf{1})$ under $SO(4) \cong SU(2)_L \times SU(2)_R$. Then, the most general mass-mixing Lagrangian read [16]

$$\begin{aligned}\Delta\mathcal{L} = -m_Q \bar{Q}_L Q_R - m_T \bar{T}_L T_R - y_L^t f_\pi \, (\bar{q}_L \Delta_L^t)_I \left(a_L^t U_{Ii} Q_R^i + b_L^t U_{I5} T_R \right) \\ - y_R^t f_\pi \, (\bar{t}_R \Delta_R^t)_I \left(a_R^t U_{Ii} Q_L^i + b_R^t U_{I5} T_L \right) + \text{h.c.},\end{aligned} \tag{2.80}$$

where

$$\Delta_L^t = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -i & 0 \\ 1 & i & 0 & 0 & 0 \end{pmatrix}, \quad \Delta_R^t = -i \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.81}$$

Before EWSB, the above Lagrangian is completely invariant under $SU(2)_L \times U(1)_Y$. In the limit of $y_L^t = y_R^t = 0$, it is also invariant under the bigger symmetry $SO(5) \times U(1)_X$. However, we can restore the $SO(5)$ invariance even in the limit of non-vanishing linear mixings by promoting the coupling matrices $\Delta_{L,R}^t$ to spurions $\hat{\Delta}_{L,R}^t$, transforming under the global $SO(5)$ of the strong sector in the same way the corresponding resonances do. Indeed, if the $\hat{\Delta}_{L,R}^t$ transform in addition appropriately under the elementary symmetry group $SU(2)_L \times U(1)_R$ of the elementary fermions, also the linear mixings are invariant under the full global symmetry of the rest of the Lagrangian. As a consequence, also the Higgs potential needs to formally respect the $SO(5)$ symmetry (and the elementary symmetry), which should then be broken by the vevs of the spurions $\langle\hat{\Delta}^t_{L,R}\rangle = \Delta^t_{L,R}$ in order to generate a non-trivial potential. Thus the form of the Higgs potential can be constructed by forming all possible invariants under the full global symmetry, containing at least one spurion $\Delta^t_{L,R}$, set to its vev, and the Goldstone-Higgs matrix U. As the spurions are always accompanied by the linear mixing parameters $y^t_{L,R}$, taking the role of an expansion parameter, a series in powers of the spurions can be established. In order for the elementary $SU(2)_L \times U(1)_R$ symmetry to be respected, the spurions can only enter in the combinations ${\Delta^t_L}^\dagger \Delta^t_L$ and ${\Delta^t_R}^\dagger \Delta^t_R$.

For the case under consideration, the form of the potential at $\mathcal{O}(\Delta^2)$ is thus fixed to

$$V_2(h) = \frac{N_c m_\psi^4}{16\pi^2}\left[\frac{{y^t_L}^2}{g_\psi^2}\, c^t_L\, v_L^{(5)}(h) + \frac{{y^t_R}^2}{g_\psi^2}\, c^t_R\, v_R^{(5)}(h)\right], \tag{2.82}$$

where the prefactors follow from naive dimensional analysis and the fact that quarks enter in $N_c = 3$ colors. m_ψ is the general mass scale for the first fermionic resonances whereas $g_\psi \equiv m_\psi/f_\pi$. The concrete values for the coefficients $c_{L,R}$, which are generically of $\mathcal{O}(1)$, need to be determined from an explicit calculation and cannot be fixed by symmetries alone. Nevertheless, the $SO(5)$ symmetry already tells us that the Higgs field can only enter in two structures at this order [17]

$$\begin{aligned} v_L(h) &= \left(U^T {\Delta^t_L}^\dagger \Delta^t_L U\right)_{55} = \frac{1}{2}\sin^2(h/f_\pi)\,, \\ v_R(h) &= \left(U^T {\Delta^t_R}^\dagger \Delta^t_R U\right)_{55} = \cos^2(h/f_\pi) = 1 - \sin^2(h/f_\pi)\,, \end{aligned} \tag{2.83}$$

where we have employed (2.81) and the explicit form of the Goldstone matrix. We inspect that, since the constant term in the second line can be neglected in the Higgs potential, only one functional dependence on the Higgs field is present. The combinations of spurions exhibit a block-diagonal structure and do not mix the fifth component of $U_{I5} = \Sigma$ with the other four components. In consequence (dropping a constant), we get

$$V_2(h) \cong \frac{N_c m_\psi^4}{16\pi^2 g_\psi^2}\left[c^t_L \frac{{y^t_L}^2}{2} - c^t_R\, {y^t_R}^2\right]\sin^2(h/f_\pi)\,. \tag{2.84}$$

This leading contribution to the potential does however not yet lead to a viable phenomenology. Its minimum is realized for $h/f_\pi = 0, \frac{\pi}{2}, \ldots$, which means that we can not have a realistic symmetry breaking with $0 < v < f_\pi$. To fix this problem we need to take into account formally subleading contributions. While no new independent $SO(5)$ invariant structures appear at $\mathcal{O}(\Delta^4)$, one can have products of the structures (2.83) which lead to a different trigonometric dependence on h,

$$\begin{aligned}
V_4(h) &= \frac{N_c m_\psi^4}{16\pi^2}\left[\frac{y_L^{t\,4}}{g_\psi^4}\,c_{LL}^t\,[v_L(h)]^2 + \frac{y_R^{t\,4}}{g_\psi^4}\,c_{RR}^t\,[v_R(h)]^2 + \frac{y_L^{t\,2}y_R^{t\,2}}{g_\psi^4}\,c_{LR}^t\,v_L(h)\,v_R(h)\right] \\
&\cong \frac{N_c m_\psi^4}{16\pi^2 g_\psi^4}\left[\left(c_{LL}^t\frac{y_L^{t\,4}}{4} - c_{RR}^t y_R^{t\,4}\right)\sin^2(h/f_\pi)\right. \qquad (2.85)\\
&\qquad \left. - \left(c_{LL}^t\frac{y_L^{t\,4}}{4} + c_{RR}^t y_R^{t\,4} - c_{LR}^t\frac{y_L^{t\,2}y_R^{t\,2}}{2}\right)\sin^2(h/f_\pi)\cos^2(h/f_\pi)\right].
\end{aligned}$$

In particular, defining generally

$$V(h) = V_2(h) + V_4(h) \equiv \alpha\sin^2(h/f_\pi) - \beta\sin^2(h/f_\pi)\cos^2(h/f_\pi)\,, \qquad (2.86)$$

we naturally obtain

$$\alpha \sim y_{L,R}^{t\,2}/g_\psi^2, \qquad \beta \sim y_{L,R}^{t\,2}y_{L,R}^{t\,2}/g_\psi^4. \qquad (2.87)$$

In order to allow for a viable EWSB, the leading contribution to α, originating from $V_2(h)$, needs to feature a tuning within its contributions that brings it from its natural size of $\mathcal{O}(y_{L,R}^{t\,2}/g_\psi^2)$ down to $\mathcal{O}(y_{L,R}^{t\,4}/g_\psi^4)$. Explicitly, the (non-trivial) minimum of the potential (2.86) occurs at

$$\sin^2(h/f_\pi) = \frac{\beta-\alpha}{2\beta}\,, \qquad (2.88)$$

which requires $\alpha-\beta$ to be as small as

$$\alpha - \beta = -2\beta\sin^2(v/f_\pi) \qquad (2.89)$$

in order to allow for the sought solution. Comparing this required size to its natural size of $\alpha-\beta\sim y_t^2/g_\psi^2$, we obtain the famous "double tuning" [16]

$$\Delta^{-1} \sim \frac{y_t^4/g_\psi^4\sin^2(v/f_\pi)}{y_t^2/g_\psi^2} = \frac{y_t^2}{g_\psi^2}\sin^2(v/f_\pi)\,, \qquad (2.90)$$

i.e., the coefficients entering $V(h)$ need not only to cancel to $\sim\sin^2(v/f_\pi)y_t^2/g_\psi^2 \ll y_t^2/g_\psi^2$ (the standard tuning due to $v \ll f$), but another tuning in the contributions to V_2 is required to make it also one order smaller in y_t^2/g_ψ^2. Moreover we observe that

$$\begin{aligned}
m_H^2 &= \frac{8}{f_\pi^2}\cos^2(v/f_\pi)\sin^2(v/f_\pi)\beta \\
&= f_\pi^2\frac{N_c}{2\pi^2}\left(\frac{c_{LL}^t}{4}y_L^{t\,4} - \frac{c_{LR}^t}{2}y_L^{t\,2}y_R^{t\,2} + c_{RR}^t y_R^{t\,4}\right)\cos^2(v/f_\pi)\sin^2(v/f_\pi)\,,
\end{aligned} \qquad (2.91)$$

and thus

$$m_H \sim \sqrt{\frac{3}{2\pi^2}}\,y_t^2\,v\,. \qquad (2.92)$$

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