Exercise 1. ***Compton scattering***

Consider the scattering process $e^-(k_1)\gamma(k_2) \to e^-(p_1)\gamma(p_2)$. To order e_0^2 (i.e. the lowest non-trivial order) the matrix element for this process can be written as

$$\begin{aligned}\langle p_1, p_2|S|k_1, k_2\rangle\Big|_{e_0^2} &= \langle p_1, p_2|T\big(e^{-ie_0 \int d^4y\, \bar\psi \not{A} \psi}\big)|k_1, k_2\rangle\Big|_{e_0^2} \\ &= \frac{(-ie_0)^2}{2}\langle p_1, p_2|T\Big(\int d^4y\, \bar\psi(y)\not{A}(y)\psi(y) \int d^4x\, \bar\psi(x)\not{A}(x)\psi(x)\Big)|k_1, k_2\rangle\end{aligned}$$

where $|k_1, k_2\rangle \equiv b^\dagger_{k_1} a^\dagger_{k_2}|0\rangle$ and $\langle p_1, p_2| \equiv \langle 0|b_{p_1} a_{p_2}$ are the free initial and final states with an electron and a photon.

Starting from this expression evaluate the matrix element directly (without applying Feynman rules) taking into account that only connected parts contribute. What does change if we replace the electrons by positrons?

Exercise 2. $e^+\, e^- \to \mu^+\, \mu^-$

Draw all one-loop Feynman diagrams (i.e. all diagrams of order e_0^4) contributing to the matrix element of the process $e^+(k_1)\, e^-(k_2) \to \mu^+(p_1)\, \mu^-(p_2)$.

Apply the Feynman rules to get an analytic expression for the various diagrams (you are not asked to evaluate these expressions!).

Exercise 3. ϕ^3 ***theory***

In Exercise 3 of Sheet 7 you considered all contributions to the 4-point function for a scalar theory with an interaction

$$\mathcal{H}_I = \frac{\lambda_0}{3!}\,\phi^3$$

Repeat this exercise with $:\mathcal{H}_I:$, the normal-ordered interaction Hamiltonian. How do the results differ between using $\mathcal{H}_I$ and $:\mathcal{H}_I:$?