Exercise 1. *Weyl spinors*

We write a Lorentz transformation in terms of the six parameters $\vec{\theta}$ and $\vec{\eta}$ as $\Lambda^{\mu}{}_{\nu} = g^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$ with

$$\omega^{\mu}{}_{\nu} = \begin{pmatrix} 0 & \eta_1 & \eta_2 & \eta_3 \\ \eta_1 & 0 & -\theta_3 & \theta_2 \\ \eta_2 & \theta_3 & 0 & -\theta_1 \\ \eta_3 & -\theta_2 & \theta_1 & 0 \end{pmatrix} \tag{1}$$

(a) Write the transformation property of a (contravariant) 4-vector under infinitesimal rotations as well as under infinitesimal boosts in terms of the (infinitesimal) parameters $\vec{\theta}$ and $\vec{\eta}$.

(b) Using the transformation property of a left-handed Weyl spinor ψ_L, show that $\psi_L^{\dagger}\bar{\sigma}^{\mu}\psi_L$ transforms as a (contravariant) 4-vector.

(c) Deduce from your working in (b) that $\psi_R^{\dagger}\sigma^{\mu}\psi_R$, where ψ_R is a right-handed Weyl spinor, also transforms as a (contravariant) 4-vector.

Exercise 2. *Poincare Algebra*

In the lecture it was shown that the the generators $J^{\mu\nu}$ transform as

$$D(\Lambda, a) J^{\mu\nu} D^{-1}(\Lambda, a) = \Lambda_{\sigma}{}^{\mu}\Lambda_{\kappa}{}^{\nu}\left(J^{\sigma\kappa} + a^{\kappa}P^{\sigma} - a^{\sigma}P^{\kappa}\right) \tag{2}$$

under a general Poincare transformation $x^{\mu} \to \Lambda^{\mu}{}_{\nu}x^{\nu} + a^{\mu}$.

(a) Considering a pure translation $\Lambda^{\mu}{}_{\nu} = g^{\mu}{}_{\nu}$ with infinitesimal a^{μ} derive the commutation relations between $J^{\mu\nu}$ and P^{ρ}.

(a) Considering a homogeneous Lorentz transformation $a^{\mu} = 0$ and $\Lambda^{\mu}{}_{\nu} = g^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$ with infinitesimal $\omega^{\mu}{}_{\nu}$ derive the commutation relations between $J^{\mu\nu}$ and $J^{\rho\sigma}$.

Exercise 3. *Dirac equation*

The most general linear (in ∂_t and $\vec{\nabla}$) wave equation can be written as $(i\,\partial^{\mu}\gamma_{\mu} - m)\psi = 0$. Show that the requirement that this equation is consistent with the Klein-Gordon equation entails that the "coefficients" γ_{μ} satisfy the relations $\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$.

Hence show that the coefficients γ_{μ} have to be at least 4×4 matrices.