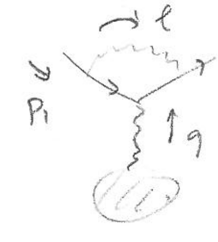
Addendum: IR singularities

after renormalization, off-shell Green functions are finite

however, going on shell (for S-matrix elements)

$\rightarrow$ infrared singularities (IR) possible!

example 

$$\sim \int \frac{d^D\ell}{(2\pi)^D} \frac{\bar u(p_2)\gamma^\nu(\not p_2 - \not\ell + m)\gamma^\mu(\not p_1 - \not\ell + m)\gamma_\nu u(p_1)}{\ell^2\,[(\ell-p_1)^2-m^2]\,[(\ell-p_2)^2-m^2]}$$

(in Feynman gauge)

consider region $\ell \to 0$

denominator: $[(\ell-p_1)^2 - m^2] = \ell^2 - 2\ell p_1 \rightsquigarrow -2\ell p_1 \qquad (p_1^2 = m^2 !!)$

numerator $\quad (\not p_1 - \not\ell + m)\gamma_\nu u(p_1) \sim (\not p_1 + m)\gamma_\nu u(p_1) = 2p_1^\nu u(p_1) + \overbrace{\gamma^\nu(-\not p_1 + m)u(p_1)}^{0}$

IR region of loop integral: $\displaystyle \int \frac{d^D\ell}{(2\pi)^D} \frac{2p_2^\nu \cdot 2p_{1\nu}\ \bar u(p_2)\gamma^\mu u(p_1)}{\ell^2\,(2\ell\cdot p_1)(2\ell\cdot p_2)}$

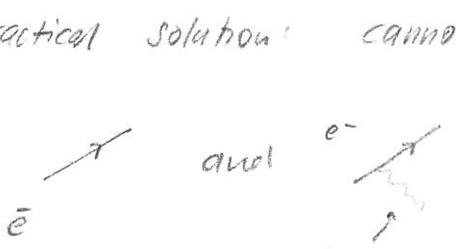
there is an IR singularity $\displaystyle \rightarrow \int_0 \frac{d^D\ell}{\ell^4} \sim \int_0 \frac{d\ell}{\ell}$

Note: IR singularity is regularized by dim reg!

The source of this "IR catastrophe" is the violation of our basic assumption in scattering theory

e^- is not "free" for $T \to \pm\infty$ due to long-range interactions (massless photon!) $\quad$ (recall QM $\lim_{r\to\infty} r\,V(r) \neq 0$!)

Practical solution: cannot distinguish between

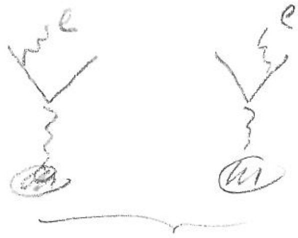


true asymptotic state is e^- + "cloud" of soft photons

(for massless e^- need also collinear photons)

Section 6.2 $\nearrow$

→ add bremsstrahlung contributions



$$\sim \bar{u}(p_2)\gamma^\mu \frac{p_1-\ell+m}{(p_1-\ell)^2-m^2}\,\varepsilon(\ell)\,u(p_1) = \frac{p_1\cdot\varepsilon}{p_1\cdot\ell}\cdot\bar{u}\gamma^\mu u$$

$$\bar{u}(p_2)\,\varepsilon\,\frac{p_2+\ell+m}{(p_2+\ell)^2-m^2}\gamma^\mu u(p_1) \to -\frac{p_2\,\varepsilon}{(p_2\ell)}\,\bar{u}\gamma^\mu u$$

add up both contributions: $\bar{u}(p_2)\gamma^\mu u(p_1)\underbrace{\left(\frac{p_1^\nu}{(p_1\cdot\ell)}-\frac{p_2^\nu}{(p_2\cdot\ell)}\right)}_{\text{eikonal factor}}\varepsilon_\nu(\ell)$

→ full contribution:

interference: virtual corrections $\sim\alpha\cdot e^2\sim\alpha^2$ real corrections $\sim\alpha\cdot e^2\sim\alpha^2$

different, but (for soft photon) indistinguishable final state

virtual IR singularities (from loop integral)

real IR sing. from phase-space int

$$\int\frac{d^3\vec{\ell}}{(2\pi)^3\cdot 2E_\ell}\;|\bar{u}\gamma^\mu u|^2\left(\frac{p_1^\nu}{(p_1\ell)}-\frac{p_2^\nu}{(p_2\ell)}\right)^2$$

$E=|\vec{\ell}|$ $\sim\int\frac{d^3\ell}{\ell^3}\to$ IR sing

Bloch Nordsieck: singularities cancel!

LO NLO virtual NLO real

$$\int\frac{d^D\ell}{(2\pi)^D}\frac{2\times(p_1p_2)}{\ell^2(\ell p_1)(\ell p_2)}|\bar{u}\gamma^\mu u|^2 + \int\frac{d^{D-1}\vec{\ell}}{(2\pi)^{D-1}2E_\ell}\frac{-2\,p_1\cdot p_2}{(p_1\ell)(p_2\ell)}|\bar{u}\gamma^\mu u|^2$$

can compute IR safe observables (indep. on emission of soft photon)