Sheet II

Return by 3.10.2013

Question 1 [*S_3 and D_3*]: The symmetric group in 3 elements, S_3, is generated by two group elements σ_1 and σ_2 that satisfy the relations

$$\sigma_1^2 = \sigma_2^2 = e\,, \qquad \sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2\,. \tag{1}$$

Show that S_3 is isomorphic to D_3 as defined in the lecture (or on Sheet I).

Hint: To construct the isomorphism, define its action on the generators and show that this can be extended to a well-defined and bijective group homomorphism.

Question 2 [*Dihedral group — Part II*]: Find all the irreducible representations of the dihedral group D_n, distinguishing between the cases $n = 2l$ and $n = 2l + 1$.

Hints: All irreducible representations of D_n have dimension 1 or 2. Remember that we have the relation

$$|G| = \sum_{i=1}^{m} (\dim V_i)^2 \ ,$$

where the sum runs over the inequivalent irreducible representations of the finite group G. Find enough inequivalent irreducible representations such that this formula is satisfied.

Question 3 [*Irreducible representations of abelian groups*]: Show that the irreducible representations of finite abelian groups are all one-dimensional[1].

Hint: Recall Schur's Lemma: Suppose V, W are irreducible representations of G and $T : V \to W$ is a G-module homomorphism, i.e.

$$T \circ \rho_V = \rho_W \circ T \ , \tag{2}$$

then

(i) either T is an isomorphism or $T = 0$;

(ii) if $V = W$, then $T = \lambda \cdot \mathbf{1}$ for some $\lambda \in \mathbb{C}$, where $\mathbf{1}$ is the identity map.

[1] Actually, this result also holds for infinite abelian groups.