Statistical Physics
Exercise 1

HS 13
Prof. M. Sigrist

Exercise 1.1 Classical ideal paramagnet

We consider an ideal paramagnet of magnetic moments in a magnetic field. The magnetic moments only have two orientations, parallel and antiparallel to the magnetic field. The Hamiltonian of the system is given by

$$\mathcal{H} = -\sum_{i=1}^{N} m_i H \ , \tag{1}$$

with $m_i = \pm m$, H the magnetic field and N the number of magnetic moments.

a) Calculate the internal energy, entropy, magnetization and magnetic susceptibility using the micro-canonical ensemble. *Hint:* Use combinatoric relations for binomial systems to determine the micro-canonical phase space count.

b) Calculate the internal energy, entropy, magnetization and magnetic susceptibility using the canonical ensemble.

Exercise 1.2 Classical ideal lattice gas

We consider N_1 particles on a lattice of N sites ($N = N_1 + N_2$), which have the condition that only one particle can occupy a site at a time. We assume that the particles have the energy E_A on N_1 sites and E_B on the other N_2 sites. Consider the situation where $N_1 < N_2$ and analyze the following situations in both the micro-canonical and canonical ensemble.

a) The energies satisfy $E_A < E_B$.

b) The energies satify $E_A > E_B$.

c) Vary the energies continuously between case a) and b).

Exercise 1.3 Classical ideal gas in a harmonic trap

We consider non-interacting classical particles in a harmonic trap described by the Hamiltonian,

$$\mathcal{H} = \sum_i \left\{ \frac{\boldsymbol{p}_i^2}{2m} + a\boldsymbol{r}_i^2 \right\} \ . \tag{2}$$

a) Assume N particles and discuss the system in the micro-canonical ensemble.

b) Assume N particles and discuss the system in the canonical ensemble.

c) Assume a constant chemical potential μ and discuss the system in the grand canonical ensemble.

Note the differences. How would you determine/define compressibility?

Office hour: Will be announced.