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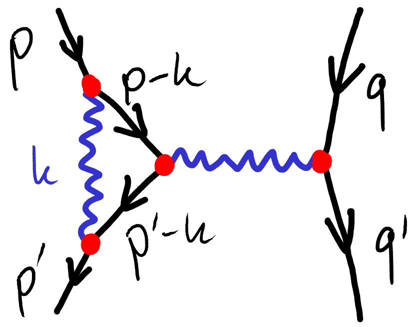
1. A one-loop correction to scattering in QED

The aim of this exercise is to gain an insight into the calculation of loop corrections to scattering amplitudes. To this end consider the one-loop corrections to $e^-e^- \to e^-e^-$ scattering in QED.

a) Draw all amputated and connected graphs that would contribute to this process. You should find 10 different contributions.

b) How does the field strength renormalisation factor for the spinors, $Z_\psi = 1 + Z_\psi^{(2)} + \ldots$, contribute at this perturbative order? How does the field strength renormalisation of the photon Z_A contribute to the process? Can you sketch suitable Feynman graphs?

Now focus on the following diagram:



c) Write the scattering matrix element corresponding to the amputated Feynman graph, and bring it to the following form

$$iM = \bar{u}(\vec{q}\,')(-ie\gamma^\mu)u(\vec{q})\,\frac{-i}{(p-p')^2 - i\epsilon}\int \frac{d^Dk}{(2\pi)^D}\,\bar{u}(\vec{p}\,')\,\frac{Z_\mu}{N'}\,u(\vec{p}). \tag{1}$$

d) Use a suitable Feynman parametrisation to rewrite the denominator N' as

$$\frac{1}{N'} = 2\int_0^1 dx \int_0^1 dy \int_0^1 dz\,\frac{\delta(1-x-y-z)}{N^3}, \tag{2}$$

where

$$N = k^2 - 2k\cdot(xp + yp') - i\epsilon. \tag{3}$$

Complete the square and show that N can be written as

$$N = k'^2 + (1-z)^2 m^2 + xy\,(p-p')^2 - i\epsilon, \qquad k' = k - xp - yp'. \tag{4}$$

$\longrightarrow$

e) Show that the numerator Z^μ can be brought to the form

$$\begin{aligned} Z^\mu = &\left(k'^2 + 2(1-4z+z^2)m^2 - 2(z+xy)(p-p')^2\right)\gamma^\mu \\ &- z(1-z)m[\gamma^\mu,\gamma^\nu](p'-p)_\nu. \end{aligned} \tag{5}$$

To do so, use:

- the anticommutation relations for γ-matrices

$$(p\cdot\gamma)\gamma^\mu = -2p^\mu - \gamma^\mu(\gamma\cdot p), \tag{6}$$

- the Dirac equation,

$$\bar{u}(\vec{p}\,')\gamma^\mu p'_\mu = m\bar{u}(\vec{p}\,'), \qquad p_\mu\gamma^\mu u(\vec{p}) = mu(\vec{p}), \tag{7}$$

- the symmetry of the integration over k', which allows the following tensorial replacements in the numerator

$$k'^\mu \to 0, \qquad k'^\mu k'^\nu \to \frac{1}{D}\eta^{\mu\nu}k'^2, \tag{8}$$

- the symmetry of the integral under the interchange $x \leftrightarrow y$,
- the Gordon identity

$$\bar{u}(\vec{p}\,')\gamma^\mu u(\vec{p}) = \frac{1}{2m}\,\bar{u}(\vec{p}\,')\left(-(p+p')^\mu - \tfrac{1}{2}[\gamma^\mu,\gamma^\nu](p'-p)_\nu\right)u(\vec{p}). \tag{9}$$

For the remainder of this problem, you may assume that the virtuality of the photon is small, $(p-p')^2 \ll m^2$.

f) Using the results obtained in exercise sheet 12, integrate over the loop momentum k'. *Note:* Split off a divergent contribution, and cut off the integral as discussed in the lecture. Can you interpret the residual dependence on the cutoff?

g) Integrate over x and y. *Note:* Cut off the integral if needed. Can you interpret the residual dependence on the cutoff?