ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Dissipation in Quantum Systems
Exercise 3

HS 12
Dr. R. Chitra

Exercise 3.1 Velocity Autocorrelation Function in Incompressible Fluids

In this exercise we want to have a closer look at the velocity autocorrelation function $\phi(t-s)$,

$$\phi(t-s) = \langle v(t)v(s)\rangle. \tag{1}$$

Assuming a drag force $F(t) \propto v(t)$ we have seen that the velocity autocorrelation function behaves for $t \to \infty$ as

$$\phi(t) \propto e^{-t/\tau}. \tag{2}$$

However, by taking more terms into account, one can show that the drag force on a hard sphere moving with velocity $v(t)$ in an incompressible fluid is given by

$$F(t) = -6\pi\eta R v(t) - \frac{2}{3}\pi\rho R^3 \dot{v}(t) - 6R^2(\pi\eta\rho)^{1/2}\int_{-\infty}^{t}(t-s)^{-1/2}\dot{v}(s)ds \tag{3}$$

with the radius R, the mass density ρ of the fluid and the viscosity η. The goal of this exercise is to comprehend that in this case, the velocity autocorrelation function $\phi(t)$ decays algebraically,

$$\phi(t) \propto t^{-3/2}, \qquad t \to \infty. \tag{4}$$

a) Print the following publication: Velocity Fluctuations of a Hard-Core Brownian Particle.[1] (If you don't have access, write an email to danmuell@itp.phys.ethz.ch .)

b) Read the paper and try to reproduce Eqn. (9)-(15) correctly!

   Hint: There are several (!) mistakes in the paper but Eqn. (13),(14),(15) are ok.

Eqn. (16)-(26) simply describe how Eq. (13) can be solved and is a purely technical business which is not interesting for us.

c) Starting with the approximate solution ψ in Eqn. (28), show that

$$\phi(t) \propto t^{-3/2} \tag{5}$$

   and conclude! Find a criterion which distinguishes systems with $\phi \propto t^{-3/2}$ and $\phi \propto e^{-t/\tau}$!

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[1] *Phys. Rev.* A 3, 13941396 (1971)