1. Regge behaviour and hard scattering limit (intermediate – hard)

We want to have a look at the features of the Veneziano amplitude

$$A_4 \sim g_\mathrm{s}\delta^{26}\left(\sum_i k_i\right)\big(I(s,t)+I(t,u)+I(u,s)\big)$$

where

$$I(s,t)=\frac{\Gamma(-1-s\kappa^2)\Gamma(-1-t\kappa^2)}{\Gamma(-2-s\kappa^2-t\kappa^2)}$$

obtained on the last problem sheet by taking two interesting limits.

a) The first limit we want to take is $s\to\infty$ with t fixed. This is the so called *Regge limit.* Why does this correspond to high energy and small angle scattering? Show that in this limit $I(s,t)$ reduces to

$$I(s,t)\sim s^{1+\kappa^2 t}\Gamma(-1-\kappa^2 t).$$

Hint: Use Stirling's approximation of the Γ function $\Gamma(1+x)\approx x^x e^{-x}\sqrt{2\pi x}$.

b) The second limit is the hard scattering limit $s\to\infty$ with t/s fixed. Argue that this corresponds to high energy and fixed angle scattering. Show that the amplitude reduces to

$$A_4\sim\exp\big(-s\log(\kappa^2 s)-t\log(\kappa^2 t)-u\log(\kappa^2 u)\big).$$

Hint: It might be easier to look at the integral expression of the amplitude and attempt a saddle point approximation, but the limit can also be taken straightforwardly.

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2. Low-energy effective action (easy – intermediate)

In the string frame the low-energy effective action is given by

$$S = \frac{1}{2\kappa^2} \int d^{26}X \sqrt{-\det G(X)}\, e^{-2\Phi} \Big(R - \tfrac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 4\partial_\mu \Phi\, \partial^\mu \Phi \Big).$$

Here $G_{\mu\nu}$ is the metric, R the associated Ricci scalar, $H_{\mu\nu\lambda} = 3\partial_{[\mu} B_{\nu\lambda]}$ is the Kalb-Ramond field strength and Φ is a scalar, the dilaton field.

a) Show that the equations of motion of these fields are equivalent to the vanishing of the β functions

$$\begin{aligned}
\beta_{\mu\nu}(G) &= \kappa^2 R_{\mu\nu} + 2\kappa^2 \nabla_\mu \nabla_\nu \Phi - \frac{\kappa^2}{4} H_{\mu\lambda\sigma} H_\nu{}^{\lambda\sigma}, \\
\beta_{\mu\nu}(B) &= -\frac{\kappa^2}{2} \nabla^\lambda H_{\lambda\mu\nu} + \kappa^2 \nabla^\lambda \Phi\, H_{\lambda\mu\nu}, \\
\beta(\Phi) &= -\frac{\kappa^2}{2} \nabla^2 \Phi + \kappa^2 \nabla_\mu \Phi \nabla^\mu \Phi - \frac{\kappa^2}{24} H_{\mu\nu\lambda} H^{\mu\nu\lambda}.
\end{aligned}$$

b) The kinetic energy term of the dilaton in the action seems to have the wrong sign. Explain why this is not so.