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hep-ph/040711

Little Higgs Models

Basic idea: Higgs doublett arises as Pseudo-Goldstone boson in the breaking of a larger continuous global symmetry (not gauge symmetry)

recall: Goldstone boson:
exactly massless, only derivative coupling
associated with breaking of exact sym.

Pseudo-Goldstone boson:
particle behaving similarly to a Goldstone boson, only derivative coupling

- $U(1)$ Goldstone boson

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi^* \phi)$$ inv. under $U(1)$: $\phi \to e^{i\alpha} \phi$

if ϕ has vev. f:

$$\phi(x) = \frac{1}{\sqrt{2}} \left(f + r(x) \right) e^{i \frac{\Theta(x)}{f}}$$

Θ: GB mode

r: radial mode

transformation under $U(1)$: $\Theta \to \Theta + \alpha$

after integrating out $r(x)$:

- $\mathcal{L}_{eff}$ contains no mass term for $\Theta(x)$, and only derivative couplings

- global symmetry breaking $SU(N) \to SU(N$

one GB per broken generator

$$(N^2-1) - [(N^2-1)-1] = 2N-1$$

through vev of $\phi \in SU(N)_{fund.}$;

parametrize as

$$\phi = \exp\left\{\frac{i}{f}\begin{pmatrix} \sim \pi^0 \mathbb{1} & & \pi_1 \\ & & \pi_{N-1} \\ \pi_1^* \cdots & \pi_{N-1}^* & \pi^0/\sqrt{2} \end{pmatrix}\right\}\begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix}$$

$$= e^{i\frac{\pi}{f}}\phi_0$$

GB: π^0 real

$\vec{\pi} = (\pi_1 \ldots \pi_{N-1})$ complex

Transformation properties of GB:

a) under unbroken symmetries $SU(N-1)$

$$U_{N-1} = \begin{pmatrix} \hat{U}_{N-1} & \\ & 1 \end{pmatrix}$$

$$\phi \to U_{N-1}\phi = (U_{N-1} e^{i\pi} U_{N-1}^+) U_{N-1}\phi_0$$

$$= \exp(i U_{N-1}\pi U_{N-1}^+)\phi_0$$

$\Rightarrow \pi \to U_{N-1}\, \pi\, U_{N-1}^{+}$

$\pi^0 \to \pi^0$ singlet

and $\begin{pmatrix} 0 & \vec{\pi} \\ \vec{\pi}^{+} & 0 \end{pmatrix} \to U_{N-1} \begin{pmatrix} 0 & \vec{\pi} \\ \vec{\pi}^{+} & 0 \end{pmatrix} U_{N-1}^{+} = \begin{pmatrix} 0 & \hat{U}_{N-1}\vec{\pi} \\ \vec{\pi}^{+}\hat{U}_{N-1}^{+} & 0 \end{pmatrix}$

- such that

$\vec{\pi} \to \hat{U}_{N-1}\, \vec{\pi}$ transforms in $SU(N-1)_{fu}$

b) under broken symmetries: coset $SU(N)/SU(N$

$$\phi \to U e^{i\pi} \phi_0 = \exp\left\{ i \begin{pmatrix} 0 & \vec{\alpha} \\ \vec{\alpha}^{+} & 0 \end{pmatrix} \right\} \exp\left\{ i \begin{pmatrix} 0 & \vec{\pi} \\ \vec{\pi}^{+} & 0 \end{pmatrix} \right\} \phi_0$$

$$= \exp\left\{ i \begin{pmatrix} 0 & \vec{\pi}' \\ \vec{\pi}'^{+} & 0 \end{pmatrix} \right\} \underbrace{U_{N-1}(\vec{\alpha}, \vec{\pi})\, \phi_0}_{= \phi_0}$$

$$= \exp\left\{ i \begin{pmatrix} 0 & \vec{\pi}' \\ \vec{\pi}'^{+} & 0 \end{pmatrix} \right\}$$

$\vec{\pi}' = \vec{\pi}'(\vec{\pi}, \vec{\alpha})$ to linear order

$\vec{\pi}' = \vec{\pi} + \vec{\alpha}$

again $\vec{\pi}$ massless, only derivative interactions

effective Lagrangian: from most general $SU(N)$-invariant function of $\phi = e^{i\frac{\pi}{f}} \phi_0$, expanded in $\frac{\partial_\mu}{\Lambda}$

$\mathcal{L} = \text{const} + f^2 |\partial_\mu \phi|^2 + O(\frac{1}{\Lambda})$

effective lagrangian:

from most general $SU(N)$-invariant function of $\phi = e^{i\frac{\pi}{f}}\phi_0$

$\rightarrow$ can contain only $SU(N)$ singlets:

$1 \in (N \otimes \bar{N}) \quad \rightarrow \phi^\dagger \phi = f$

$1 \in (\underbrace{N \otimes N \otimes \ldots \otimes N}_{N \text{ times}}) \rightarrow \epsilon^{i_1 \ldots i_N} \phi_{i_1} \ldots \phi_{i_N}$

- introducing gauge interactions

consider simplest case $SU(3) \rightarrow SU(2)$

$d(SU(3)) = 8, d(SU(2)) = 3 \Rightarrow 5$ G

$$\pi = \begin{pmatrix} -\frac{\eta}{2} & 0 & \\ 0 & -\frac{\eta}{2} & h \\ & h^\dagger & \eta \end{pmatrix}$$

h: $SU(2)$ doublet

η: $SU(2)$ singlet

still: global $SU(2)$

- coupling h to $SU(2)$ gauge bosons
  $\rightarrow$ breaks $SU(3)$
- gauge $SU(3)$
  $\rightarrow$ h and η are absorbed into longitudinal degrees of freedom of $SU(3)/SU(2)$ gauge bosons

– gauge $SU(3)$ and introduce two copies of ϕ triplets with aligned vev $f \simeq O(\text{TeV})$

$$\phi_1 = e^{i\pi_1/f}\begin{pmatrix}0\\0\\f\end{pmatrix} \qquad \phi_2 = e^{i\pi_2/f}\begin{pmatrix}0\\0\\f\end{pmatrix}$$

kinetic Lagrangian

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2$$

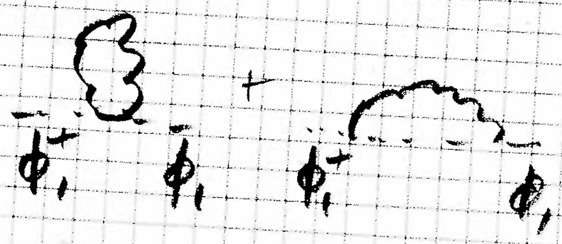
→ only one linear combination of π_1 and π_2 is absorbed into $SU(3)/SU(2)$

Parametrization:

$$\phi_1 = \exp\left(i\begin{pmatrix} & h\\ h^\dagger & \end{pmatrix}\right)\exp\left(i\begin{pmatrix} & h\\ h^\dagger & \end{pmatrix}\right)\begin{pmatrix}0\\f\end{pmatrix}$$

$$\phi_2 = \underbrace{\exp\left(i\begin{pmatrix} & h\\ h^\dagger & \end{pmatrix}\right)}_{\text{GB of } SU(3)/SU(2)}\underbrace{\exp\left(-i\begin{pmatrix} & h\\ h^\dagger & \end{pmatrix}\right)}_{\text{pseudo-GB}}\begin{pmatrix}0\\f\end{pmatrix}$$

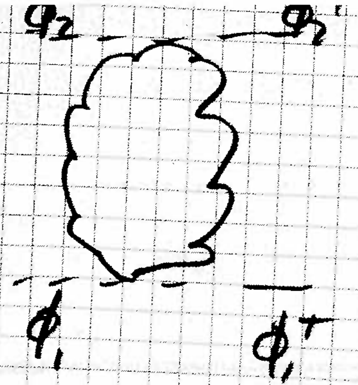
Higgs mass and Higgs potential generated from radiative corrections (gauge ~~boson~~ loops)



$$= \frac{g^2}{16\pi^2}\,\overset{\text{cut-off, quadratic div.}}{\Lambda^2}\,(\phi_1\phi_1^\dagger)_{33}$$

$$= \frac{g^2}{16\pi^2}\Lambda^2 f^2$$

→ does not involve h, only corrects ...



$$= \frac{g^4}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)|\phi_1^\dagger\phi_2|^2$$

use: $\phi_1^\dagger\phi_2 = (0\ f)\exp\left\{-\frac{2i}{f}\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right\}\begin{pmatrix}0\\ f\end{pmatrix}$

$$= \left(f^2\begin{pmatrix}1 & \\ & 1\end{pmatrix} - 2i\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix} - 2\begin{pmatrix} hh^\dagger & \\ & h^\dagger h\end{pmatrix}\right)_{33}$$

$$= f^2 - 2h^\dagger h + \dots$$

$\Rightarrow$ generates mass term for h:

$$\frac{g^2}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)f^2 \ll f^2$$

- free of quadratic ~~one~~-loop divergence
- mass scales: f: vev of $SU(3)/SU(2)$ $\mathcal{O}(\text{TeV})$

  $\frac{f}{4\pi}$: "weak mass"

  $\Lambda \sim 4\pi f$: cut-off scale

"The simplest little Higgs model"

- gauge boson masses: use $SU(3)_w \otimes U(1)_X \to SU(2)_L \otimes U(1$

use $\left| \partial_\mu + i g A_\mu^a T^a - \frac{1}{3} g_x A_\mu^x \right) \phi_i \right|^2$

$$\xrightarrow[\text{vev}]{} \mathrm{tr}\left[\left(g A_\mu^a T^a - \frac{1}{3} g_x A_\mu^x \right) \phi_i \phi_i^+ \right]$$

and $\langle \phi_1 \phi_1^+ + \phi_2 \phi_2^+ \rangle = \begin{pmatrix} \langle h h^+ \rangle & 0 \\ 0 & f^2 \end{pmatrix}$

- with ew-symmetry breaking vev:

$$\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

one obtains:

$SU(2)$ $\quad m_{W^\pm}^2 = \frac{g^2 v^2}{4}$ as in SM

$SU(3)/SU(2)$ $\quad m_{W^{\pm\prime}}^2 = m_{W^{0\prime}}^2 = \frac{g^2}{2} f^2$

- neutral gauge bosons:

two diagonal $SU(3)$ generators T_3, T_8

$SU(2)$: $W_\mu^3 = A_\mu^3$

$$B_\mu = \frac{-g_x A_\mu^8 + \sqrt{3} g B_\mu^x}{\sqrt{3g^2 + g_x^2}} \qquad U(1)_Y \text{ gauge bos}$$

$$Z_\mu' = \frac{\sqrt{3} g A_\mu^8 + g_x B_\mu^x}{\sqrt{3g^2 + g_x^2}}$$

$\Rightarrow U(1)_Y$ coupling $\quad g' = g_x / \sqrt{1 + \frac{g_x^2}{3g^2}}$

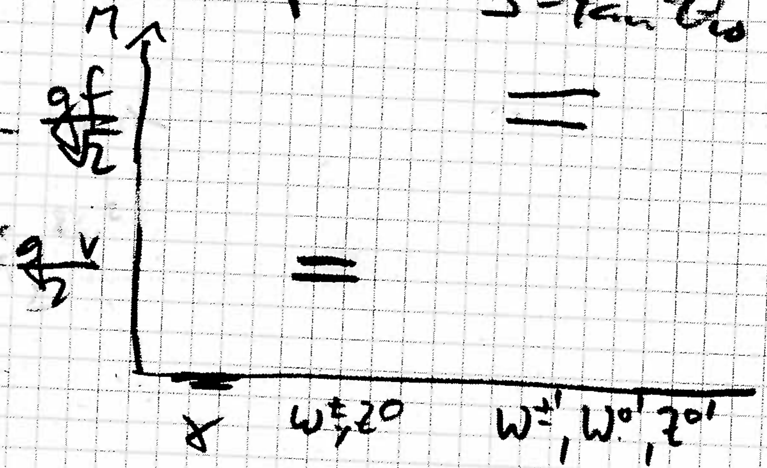
after $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$

$$\Rightarrow \quad m_\gamma^2 = 0$$

$$m_Z^2 = \frac{g^2 v^2}{4}\left(1+\tan^2\theta_W\right) \leftarrow \text{as in}$$

$$m_{Z'}^2 = g^2 f^2 \, \frac{2}{3-\tan^2\theta_W}$$

Spectrum



Matter is embedded in $(SU(3)_C, SU(3)_W)_{U(1)_X}$ multiplets

	Quarks	Leptons
left-handed	$\psi_Q = (3,3)_{\frac{1}{3}}$	$\psi_L = (1,3)_{-\frac{1}{3}}$
right-handed	$d^c = (\bar{3},1)_{\frac{1}{3}}$	$e^c = (1,1)_1$
	$2\times u^c = (\bar{3},1)_{-\frac{2}{3}}$	$\nu^c = (1,1)_0$

Symmetry breaking through vev of two complex scalar triplett fields

$$\phi_1, \phi_2 = (1,3)_{-\frac{1}{3}}$$

Yukawa-terms in this model (-> fermion masses

$\lambda_1^u u_1^c \phi_1^+ \psi_Q$ u-type quarks

$+ \lambda_2^u u_2^c \phi_1^+ \psi_Q$ heavy partners of u-type quarks

$+ \lambda^d d^c \frac{\phi_1 \phi_2 \psi_Q}{\Lambda}$ d-type quarks

$+ \lambda^\nu \nu^c \phi_1^+ \psi_L$ neutrinos

$+ \lambda^e e^c \frac{\phi_1 \phi_2 \psi_L}{\Lambda}$ charged leptons

with $\phi_1^+ \psi_Q \rightarrow 3 \otimes \overline{3}$ singlet

$\phi_1 \phi_2 \psi_Q \rightarrow 3 \otimes 3 \otimes 3$ singlet

$d(SU(3)) = 8$, $d(SU(2)) = 3$ -> 5 broken generators