Estimation of oblique electroweak corrections

(Peskin & Takeuchi)

We have great experimental evidence that the $SU(2)_L \otimes U(1)_Y$ gauge theory describes correctly electroweak interactions. However, the Higgs sector which gives W & Z bosons their masses is a mystery. There are many hypotheses & extensions of the Standard Model but so far, including the minimal SM theory, all of those have not been verified by a discovery of the particle of particles which they predict.

We can test indirectly the Higgs sector with precision measurements of the parameters of the weak interaction. General models of the Higgs sector allow large deviations from the minimal SM. However, as we have seen last week, the minimal SM has a very succesful prediction. At three level,

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\vartheta_W} = 1$$

and it is satisfied to a better than 1% precision. This restricts the possibilities for alternative Higgs sectors. Nonetheless, a large class of models which respect an approximate (Yukawa couplings & gauge couplings → 0) ~~predict the same~~ "custodial" $SU(2)_L \otimes SU(2)_R$ symmetry predict the same ρ at tree level. Custodial symmetry requires that φ^+ & φ are treated on a ~~sym~~ the same footing and can be interchanged.

OPERATOR	CUSTODIAL SYMMETRY
$(D_\mu \varphi^+)(D^\mu \varphi)$	✓
$(\varphi^+ \varphi)$	✓
$(\varphi^+ D_\mu \varphi)$	×

We also find large deviations from the minimal Standard Model if we extend the $SU(2) \times U(1)$ gauge group. This will require a mixing of the Z boson with new Z's. ~~For a non-tuned value of the mixing~~. We shall restrict our search of new physics to models with a custodial symmetry

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad @ \text{ tree-level}$$

and where the $SU(2)_L \otimes U(1)_Y$ group is the correct group for electroweak interactions for quite ~~a~~ a lot above the EWK scale M_W.

For Higgs sectors with weak interactions, we can employ perturbation theory in order to compute and analyze the impact of radiative corrections to measured parameters. We shall see that we can also analyze the impact of strong sectors too, besides not being able to use a perturbative approach, and that Goldstone bosons do not arise perturbatively either.

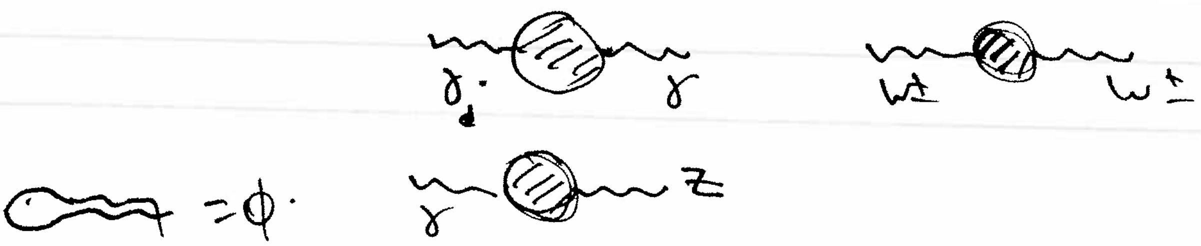
Formalism of oblique corrections

We consider weak interaction processes with only light fermions as external particles. For example,

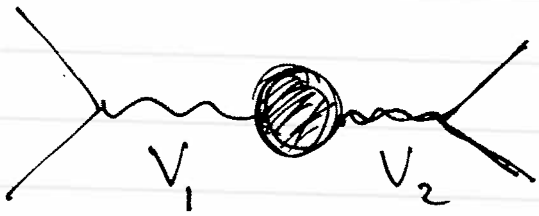
$$e^+e^- \to e^+e^-, \mu^+\mu^-, b\bar{b},$$
$$q\bar{q} \to e^+e^-, \mu^+\mu^-, e\nu,$$

etc.

These processes involve the propagation of electroweak gauge bosons

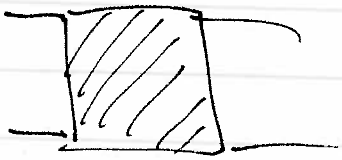


We find three types of contributions to four-fermion interactions.



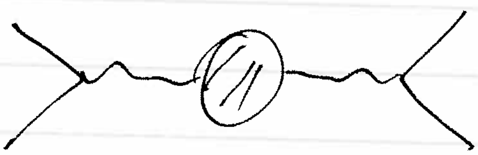
Bubble

~ Triangles

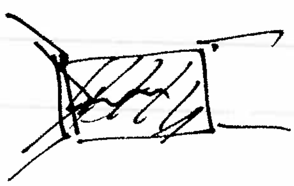


— Boxes.

We are interested to contributions due to new physics. Given that the external particles are light, the relative contributions of the three components scale as



 ~  $\frac{1}{M_{NP}^2}$

 ~  $\frac{1}{M_{NP}^4}$.

The dominant New-Physics corrections arise from self-energy corrections to the propagators. These corrections are called "oblique" due to reasons that we shall explain shortly.

<u>Notation</u>: The electroweak currents are:

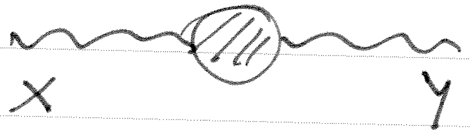
$J_Q^\mu \quad \to$ electromagnetic

$$\left.\begin{array}{l} J_3^\mu \\ J_\pm^\mu \end{array}\right\} \text{ weak isospin}$$

and $$\mathcal{L} \ni \frac{e}{\sqrt{2}\, s} \left(W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu + \right.$$

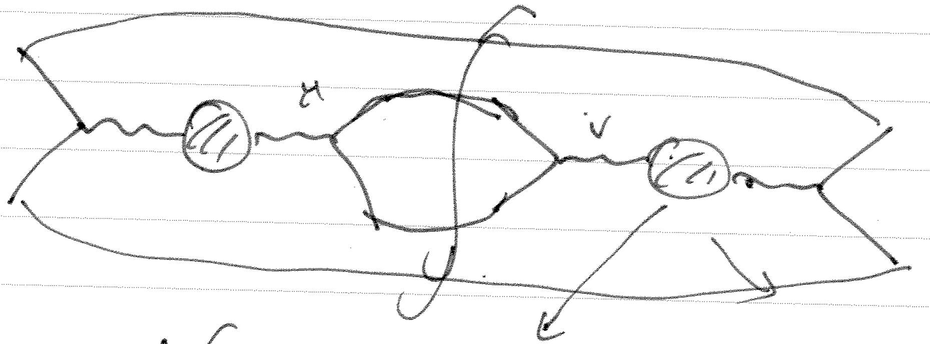
$$\left. + \frac{e}{sc} Z_\mu \left(J_3^\mu - s^2 J_Q^\mu \right) + e A_\mu J_Q^\mu \right.$$

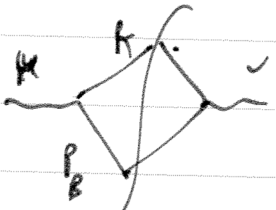
In a four-fermion process the propagator



$$= G_{XY}\, g^{\mu\nu} + $$
$$+ \left(p^\mu p^\nu \text{ terms} \right)$$

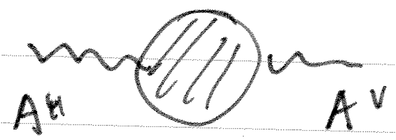
contributes to the amplitude only with its only throug its $g_{\mu\nu}$ coefficient.



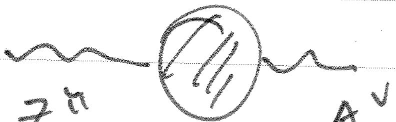
$\rightarrow$  $\sim (p_\alpha + p_b)^\mu \, (p_\alpha + p_\beta)^\nu \cdot \; p_\alpha^\mu p_\beta^\nu \to \phi$

$\mathrm{tr}(\not p \, \gamma^\mu \, p_L \gamma_\nu)^\beta$

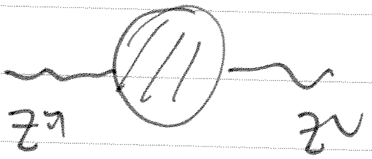
We have four propagators (when including higher orders)



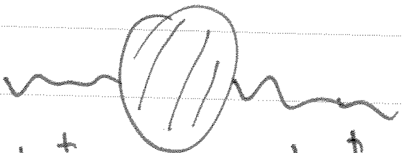
$= G_{AA} \, g^{\mu\nu} + \dots$



$= G_{ZA} \, g^{\mu\nu} + \dots$



$= G_{ZZ} \, g^{\mu\nu} + \dots$



$= G_{WW} \, g^{\mu\nu} + \dots$

We can compute the matrix-elements of the charged and neutral currents interactions

$$\mathcal{M}_{NC} = e^2 Q Q' G_{AA} +$$

$$+ \frac{e^2}{sc} G_{ZA} \left[Q(I_3' - s^2 Q') + (I_3 - s^2 Q) Q' \right]$$

$$+ \frac{e^2}{s^2 c^2} (I_3 - s^2 Q)(I_3' - s^2 Q') G_{ZZ}$$

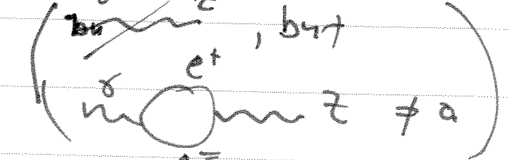
and

$$M_{CC} = \frac{e^2}{2s^2} I_+ I_- G_{WW}$$

(I_3, Q), (I_3', Q') are the electric charge and weak isospin. $I_\pm$ are weak isospin raising ~~and~~ lowering matrices

At leading order in perturbation theory

$$G^{(LO)}_{ZA} = 0 \equiv D_{ZA}$$



and $D_{BB} = G^{(LO)}_{BB} = \frac{1}{q^2 - M_{0B}^2}$, $B = A, Z, W^\pm$

$$m_{0A}^2 = 0 \quad , \quad m_{0Z}^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4}$$

$$m_{0W}^2 = \frac{e^2}{s^2} \frac{v^2}{4} \qquad \left(\left(m_{0W}^2 = m_{0Z}^2 \cdot c^2 \right) \right)$$

$s \equiv \sin\theta_W$
$c \equiv \cos\theta_W$

As we discussed, the leading modifications due to new physics arise from vacuum polarization effects. These change ~~the~~ values of G_{XY} but ~~do not~~ change the form of the M_{NC} & M_{cc} amplitudes. This is the reason that they are called "oblique" unlike triangle & box corrections which change also the form of the interactions.

Let's define the 1 particle irreducible 1PI vacuum polarization amplitudes

$$\int d^4y\, e^{-iq\cdot y} \langle J_X^\mu(y) J_Y^\nu(0) \rangle =$$

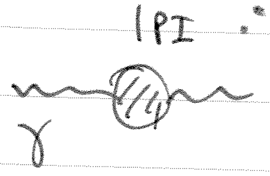
$$= i g^{\mu\nu} \Pi_{XY}(q^2) + (q^\mu q^\nu \text{ terms})$$

with $(XY) \equiv (11), (22), (33), (3Q), (QQ)$.

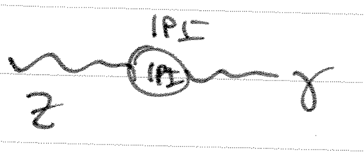
We also define:

$$\Pi_{XY}(q^2) = \Pi_{XY}(0) + q^2 \Pi'_{XY}(q^2).$$

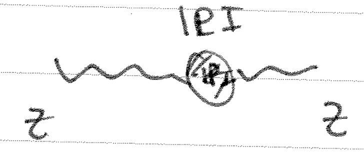
We can express



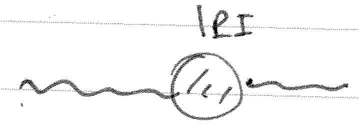
$$= \overbrace{ie^2\,\Pi_{QQ}}^{G_{QQ}}\, g^{\mu\nu} + \dots$$



$$= \frac{ie^2}{cs}\overbrace{\left(\Pi_{3Q} - s^2\Pi_{QQ}\right)}^{G_{Z\gamma}} g^{\mu\nu} + \dots$$

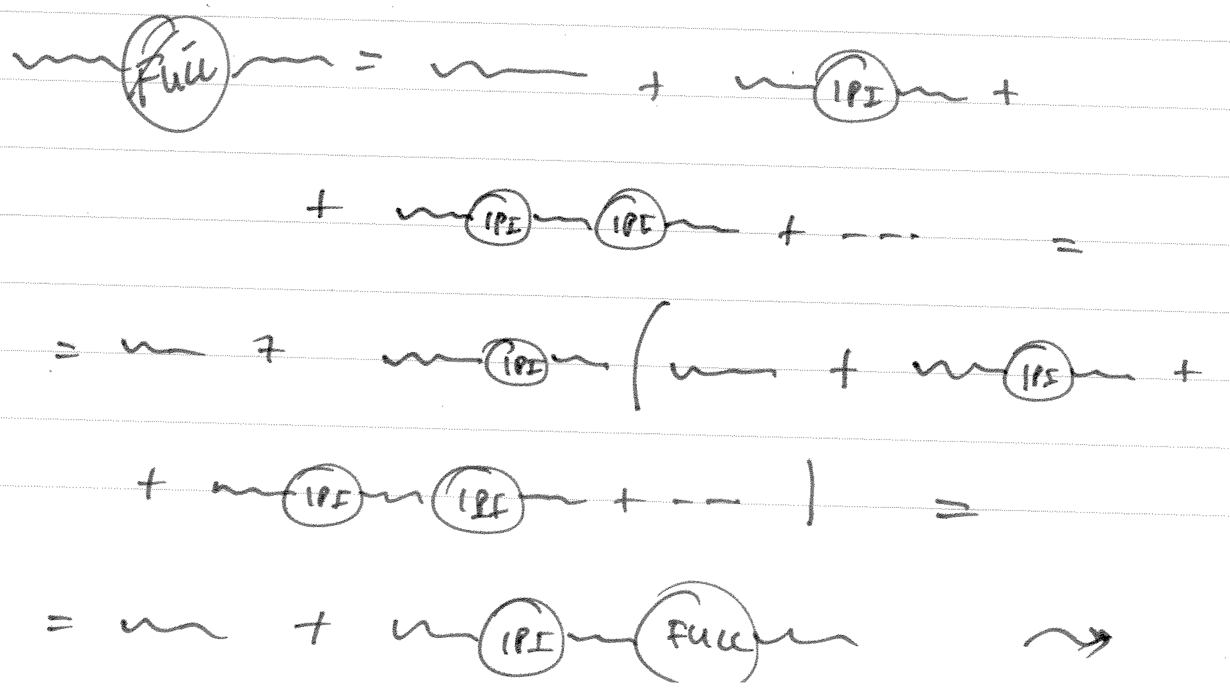


$$= i\frac{e^2}{c^2s^2}\overbrace{\left(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\,\Pi_{QQ}\right)}^{G_{ZZ}} g_{\mu\nu}$$



$$= i\frac{e^2}{s^2}\,\Pi_{11}\, g^{\mu\nu} + \dots$$

Recall that we can perform a Dyson resummation of (1PI) graphs:



$$\overset{\text{FULL}}{G_{CD}} = \overset{\text{TREE}}{D_{CD}} + \overset{\text{TREE}}{D_{CE}}\ \overset{(1PI)}{\Pi_{EF}}\ \overset{\text{FULL}}{G_{FD}} \Big),$$

After Dyson resummation, the NC and CC amplitudes become:

$$M_{NC} = \frac{e^2\, Q Q'}{q^2 - \Pi_{AA}}$$

$$+ \frac{e^2}{s^2 c^2} \frac{\left\{ I_3 - \left[s^2 - sc \frac{\Pi_{ZA}}{q^2 - \Pi_{AA}} \right] Q \right\} \left\{ I_3' - \left[s^2 - sc \frac{\Pi_{ZA}}{q^2 - \Pi_{AA}} \right] Q' \right\}}{q^2 - m_{0Z}^2 - \Pi_{ZZ} - \frac{(\Pi_{ZA})^2}{q^2 - \Pi_{AA}}}$$

$$M_{CC} = \frac{e^2}{2 s^2} \frac{I_+ \, I_-}{q^2 - m_{0W}^2 - \Pi_{WW}} .$$

~~We now have to renormalize~~

Let us define the "effective" electric charge and effective Weinberg angle:

$$e_*^2(q^2) \equiv \frac{e^2}{1 - e^2 \Pi'_{QQ}(q^2)} = e^2 \left(1 + e^2 \Pi'_{QQ}(q^2) \right)$$

$$s_*^2(q^2) = s^2 - sc\,\frac{\Pi_{ZA}(q^2)}{q^2 - \Pi'_{AA}(q^2)} =$$

$$= s^2 - e^2\left(\Pi'_{3Q}(q^2) - s^2\,\Pi'_{QQ}(q^2)\right)$$

~~Then,~~ It is OK to linearize, since we are dealing with typically small corrections. With these redefinitions of the couplings we cast the M_{NC} & M_{CC} amplitudes as:

$$M_{NC} = e_*^2\, Q\,\frac{1}{q^2}\,Q' + \frac{e^2}{s^2c^2}\cdot(I_3 - s_*^2 Q)\cdot$$

$$\cdot\,(I_3' - s_*^2 Q')\,\frac{1}{q^2 - \frac{e^2}{s^2c^2}\left[\frac{v^2}{4} + (\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ})\right]}$$

$$M_{CC} = \frac{e^2}{sc^2}\, I_+\,\frac{1}{q^2 - \frac{e^2}{s^2}\left[\frac{v^2}{4} + \Pi_{11}\right]}\, I_-$$

which maintain the same form as the Born amplitudes ~~although~~ and radiative corrections are accounted for by redefining s_θ^2, e^2 & m_{0Z}^2.

These ~~re~~definitions involve infinite quantities and we must now renormalize.
The W and Z masses are the poles of their respective propagators.

$$m_Z^2 = \frac{e^2}{s^2c^2}\frac{v^2}{4} +$$

$$+ \frac{e^2}{s^2c^2}\left(\Pi_{33} - 2s^2\,\Pi_{3Q} + s^4\,\Pi_{QQ}\right)\Big|_{q^2=m_Z^2}$$

and

$$m_W^2 = \frac{e^2}{s^2}\,v^2 + \frac{e^2}{s^2}\,\Pi_{11}(m_W^2)$$

The wave-function renormalization constants Z_Z, Z_W are the coefficients of the poles in the Z & W propagators.

$$Z_Z^{-1} = 1 - \frac{e^2}{s^2c^2}\frac{d}{dq^2}\left(\Pi_{33} - 2s^2\,\Pi_{3Q} + s^4\,\Pi_{QQ}\right)\Big|_{q^2=m_Z^2}$$

and $$Z_W^{-1} \equiv 1 - \frac{e^2}{s^2}\frac{d}{dq^2}\Pi_{11}\Big|_{q^2=m_W^2}.$$

This completes our renormalization procedure, ~~and ex~~ which expresses the amplitudes in terms of the physical pole masses.

However, we are allowed to use other finite quantities to express our amplitudes. We can use a running mass scheme and define.

$$\frac{1}{q^2 - \frac{e^2}{s^2}\left[\frac{v^2}{4} + \Pi_{11}\right]} = \frac{Z_W}{q^2 - M_{W*}^2(q^2)} .$$

~~If we did an experiment~~

$$\frac{1}{q^2 - \frac{e^2}{s^2 c^2}\left[\frac{v^2}{4} + \left(\Pi_{33}(q^2) - 2s^2 \Pi_{3Q}(q^2) + s^4 \Pi_{QQ}(q^2)\right)\right]} = \frac{Z_Z}{q^2 - M_{Z*}^2(q^2)}$$

Obviously, at $q^2 = m_W^2, m_Z^2$ we have:

$$M_{Z*}^2(m_Z^2) = m_Z^2 \quad , \quad \frac{d}{dq^2} M_{Z*}^2 \Big|_{q^2 = m_Z^2} = 0 .$$

$$M_{W*}^2(m_W^2) = m_W^2 \quad , \quad \frac{d}{dq^2} M_{W*}^2 \Big|_{q^2 = m_W^2} = 0 .$$

Finally, we define:

$$\frac{e_*^2}{S_*^2 C_*^2} Z_{Z*} = \frac{e^2}{s^2 c^2} Z_Z$$

$$\frac{e_*^2}{s_*^2} Z_{W*} = \frac{e^2}{s^2} Z_W , \qquad c_*^2 = 1 - s_*^2$$

Explicitly,

$$Z_{Z*} = Z_Z \left[1 - e^2 \Pi'_{QQ}(q^2) - \frac{e^2(c^2-s^2)}{s^2c^2} \cdot \right.$$

$$\left. \cdot \left(\Pi'_{3Q}(q^2) - s^2 \Pi'_{QQ}(q^2) \right) \right] =$$

$$= 1 + \frac{e^2}{s^2c^2} \frac{d}{dq^2} \left(\Pi_{33} - 2s^2 \Pi_{3Q} + s^4 \Pi_{QQ} \right) \Big|_{q^2 = m_Z^2} -$$

$$- \frac{e^2 s^2}{c^2} \Pi'_{QQ}(q^2) - \frac{e^2(c^2-s^2)}{s^2c^2} \Pi'_{3Q}(q^2)$$

$$Z_{W*} = Z_W \left[1 - e^2 \Pi'_{QQ}(q^2) - \frac{e^2}{s^2} \left(\Pi'_{3Q}(q^2) - s^2 \Pi'_{QQ}(q^2) \right) \right]$$

$$= 1 + \frac{e^2}{s^2} \frac{d}{dq^2} \Pi_{11} \Big|_{q^2 = m_W^2} - \frac{e^2}{s} \Pi'_{3Q}(q^2) .$$

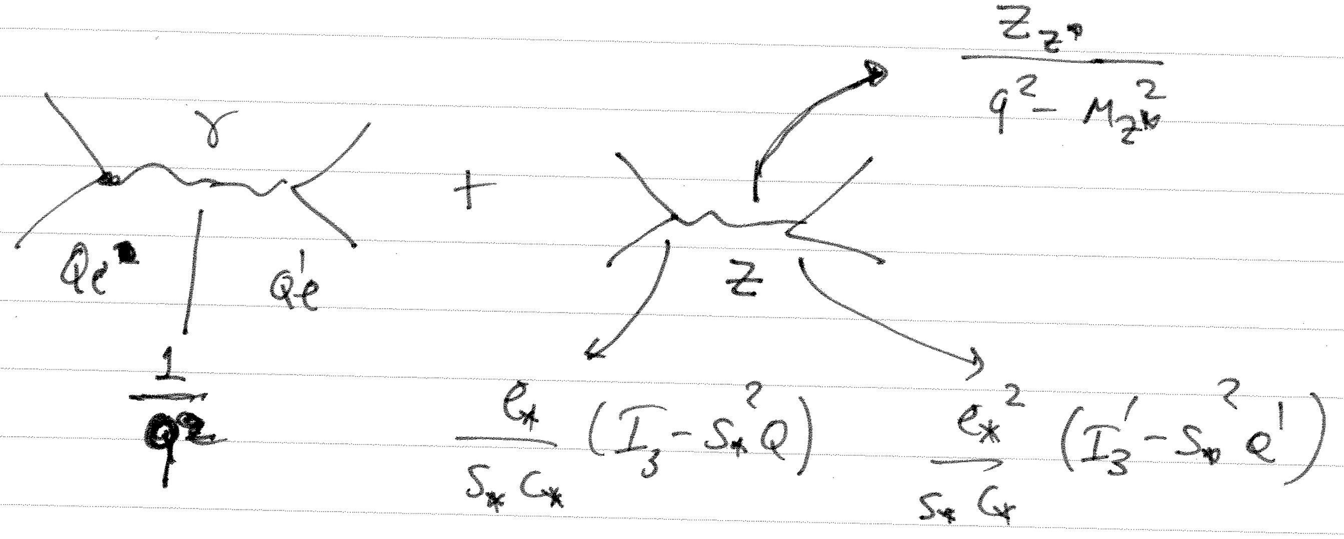
Finally,

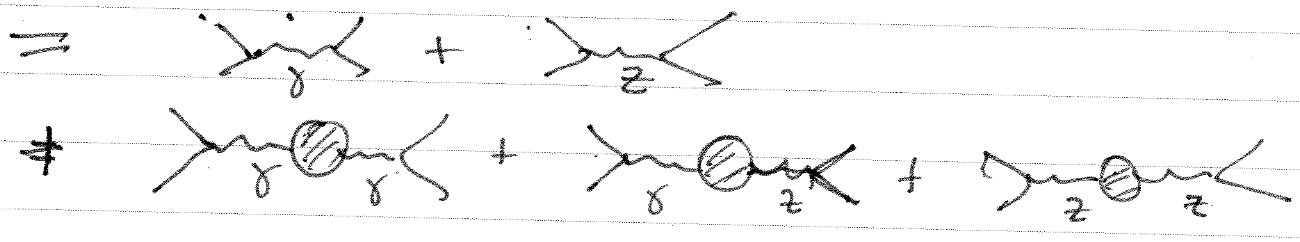
$$M_{NC} = e_*^2 Q \frac{1}{q^2} Q +$$

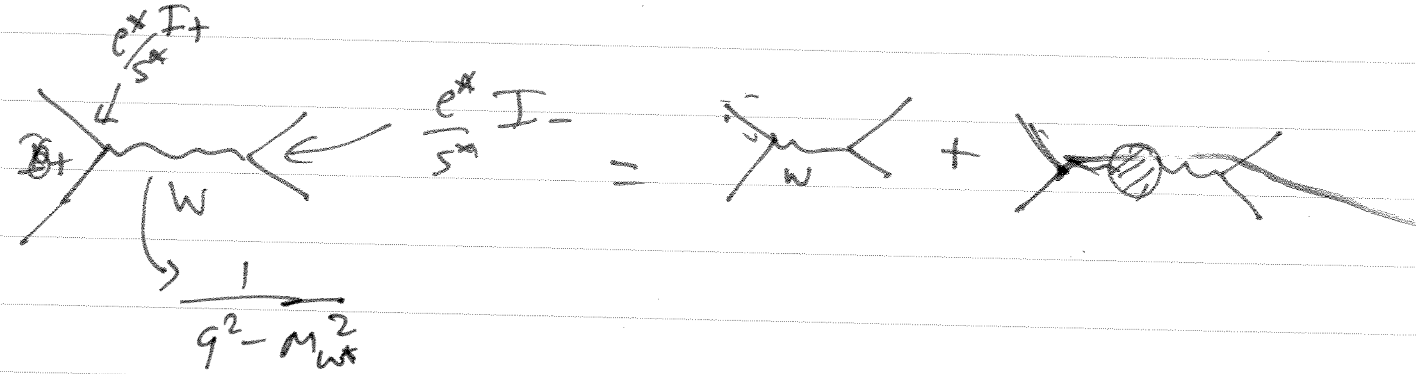
$$+ \frac{e_*^2}{s_*^2 c_*^2} (I_3 - s_*^2 Q) \frac{Z_{Z*}}{q^2 - M_{Z*}^2} (I_3' - s_*^2 Q')$$

$$M_{CC} = \frac{e_*^2}{2 s_*} I_+ \frac{Z_{W*}}{q^2 - M_{Z*}^2} I_- ,$$

All starred quantities are finite ~~but~~ energy (q^2) dependent quantities. We find that the oblique corrections are fully accounted for if in the tree-level expressions we substitute bare quantities with starred quantities.







Oblique corrections due to new physics can affect many measurements. For example, the width of the Z-boson is:

$$\Gamma_Z = Z_{Z^*} \frac{\alpha_* m_Z}{6 s_*^2 c_*^2} \sum_f \left(I_{3f} - s_*^2 q_f\right) \cdot N_f \Big|_{q^2 = m_Z^2}$$

$$N_f = N_{col} \cdot \left[1 + \frac{\alpha_s}{\pi} + \ldots \right]$$

The left-right assymmetry,

$$A_{LR}(q^2) = \frac{2\left[1 - 4 s_*^2(q^2)\right]}{1 + \left[1 - 4 s_*^2(q^2)\right]^2}$$

and so on. Note that in the SM oblique (vacuum polarization) are not always dominant. This is especially manifest for observables where photon real radiation must be accounted for, however, new physics contributions to the oblique corrections are ~~important~~ dominant, with non-oblique correction suppressed by powers of $\left(\frac{1}{m_{NP}}\right)$.

We should always check if our new models introduce ~~big~~ oblique corrections which may cause disagreements with measurements such as of Γ_Z, Γ_W, A_{LB}, etc.

At low energies, $q^2 = 0$, we obtain

$$\mathcal{M}_{NC}^{(Z)} = - \frac{\left[I_3 - s_*^2(0)\, Q\right]\left[I_3' - s_*^2(0)\, Q'\right]}{\left[\frac{v^2}{4} + \Pi_{33}(0)\right]}$$

$$\mathcal{M}_{CC} = -\frac{1}{2} I_+ \frac{1}{\left[\frac{v^2}{4} + \Pi_{11}(0)\right]} I_-$$

The same amplitudes can be obtained by using Fermi's effective theory

$$\mathcal{L}_{eff} = - \frac{4 G_F}{\sqrt{2}} \left\{ J_\mu^+ J_\mu^- + \rho_*(0) \left[J_3^\mu - s_*^2(0) J_Q^\mu\right]^2 \right\}$$

This yields:

$$\frac{1}{4\sqrt{2}\, G_F} = \frac{v^2}{4} + \Pi_{11}(0)$$

$$\frac{1}{\rho_*(0)} = \frac{\frac{v^2}{4} + \Pi_{33}(0)}{\frac{v^2}{4} + \Pi_{11}(0)} =$$

$$= 1 - 4\sqrt{2}\, G_F \left(\Pi_{11}(0) - \Pi_{33}(0) \right)$$

In summary,

- Oblique electroweak corrections may be computed from leading order matrix elements ~~via~~ by substituting

$$S \to S_*$$

$$M_Z \to M_{Z*}$$

$$M_W \to M_{W*}$$

etc

- The dependence of starred quantities on vacuum polarization graphs is relatively simple. Easy to see what is the effect of new physics on measured quantities.

- New physics may affect measurements at the lowest energies.

S, T, U Parameters

M_{CC} & M_{NC} depend ~~on S^*, T^*~~, and the starred quantities, even at lowest order depend on 3 parameters

$$s_\theta, \; v, \; e.$$

We need at least three measurements to constrain them.

$$\alpha^{-1} = 137.035989\ldots$$

$$G_F = 1.16637 \times 10^{-5}\,(\text{GeV})^{-2}$$

$$m_Z = 91.17\,\text{GeV}.$$

All starred quantities for new physics with a scale

$q^2 \sim M_Z^2 \ll M_{NP}^2$ can be

expressed linearly through S, T and U parameters

$$\alpha S = 4e^2\left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right]$$

$$\alpha T = \frac{e^2}{s^2 c^2 M_Z^2}\left(\Pi_{11}(0) - \Pi_{33}(0)\right)$$

$$\alpha U = 4e^2\left[\Pi'_{11}(0) - \Pi'_{33}(0)\right]$$

this is because for $q^2 \to 0$

$$\Pi_{QQ}(q^2) = q^2 \Pi'_{QQ}(0)$$

$$\Pi_{3Q}(q^2) = q^2 \Pi'_{3Q}(0)$$

$$\Pi_{33}(q^2) = \Pi_{33}(0) + q^2 \Pi'_{33}(0)$$

$$\Pi_{11}(q^2) = \Pi_{11}(0) + q^2 \cdot \Pi'_{11}(0)$$

we get for example:

$$\rho_*(0) = 1 + \alpha T$$

$$Z_{Z^*}(q^2) - 1 = \frac{\alpha}{4s^2c^2} S$$

$$Z_{W*}(q^2) - 1 = \frac{\alpha}{4s^2}(S+U)$$

In the above, we can assume that $\Pi_{11} \simeq \Pi_{33}$ due to custodial symmetry. So $U \sim 0$

Suppose we know the W-mass in the SM very well ~~and ha~~

Then $m_W^2 = m_W^2(\text{ref}) + \frac{\alpha c^2}{c^2 - s^2} m_Z^2 \left[-\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4 s^2} U \right]$

with new physics

precise SM-calculation.

and so on ...

Data:

S ← Z-pole assymmetries
S ← M_Z, T ← Γ_Z, α_s (from ...)
m_t ← (CDF & D0). $U \leftarrow M_W$

(IV) Examples!

Let's consider a new heavy fermion doublet (N, E)

with usual couplings to $SU(2)_L \times U(1)_Y$ and masses m_N, m_E. For $m_N, m_E \gg m_Z$,

$Y = \frac{Y_Q}{2}$

$$S = \frac{1}{6\pi}\left[1 - Y \ln\left[\frac{m_N^2}{M_E^2}\right]\right]$$

$$T = \frac{1}{16\pi s^2 c^2 m_Z^2}\left[m_N^2 + m_E^2 - \frac{2 m_N^2 m_E^2}{m_N^2 - m_E^2} \log\left(\frac{m_N^2}{m_E^2}\right)\right]$$

$$U = \frac{1}{6\pi}\left[-\frac{5m_N^4 - 22m_N^2 m_E^2 + 5m_E^4}{3(m_N^2 - m_E^2)^2} + \frac{m_N^6 - 3m_N^4 m_E^2 + 3m_N^2 m_E^4 + m_E^6}{(m_N^2 - m_E^2)^3} \log\left(\frac{m_N^2}{m_E^2}\right)\right]$$

for $\Delta m = |m_N - m_E| \ll m_N, m_E$

$$S = \frac{1}{6\pi} \qquad \left(\text{for quarks} \rightarrow \frac{N_c}{6\pi}\right)$$

$$T = \frac{1}{12\pi s^2 c^2}\left[\frac{(\Delta m)^2}{m_Z^2}\right]$$

$$U = \frac{2}{15\pi}\left[\frac{(\Delta m)^2}{m_N^2}\right]$$

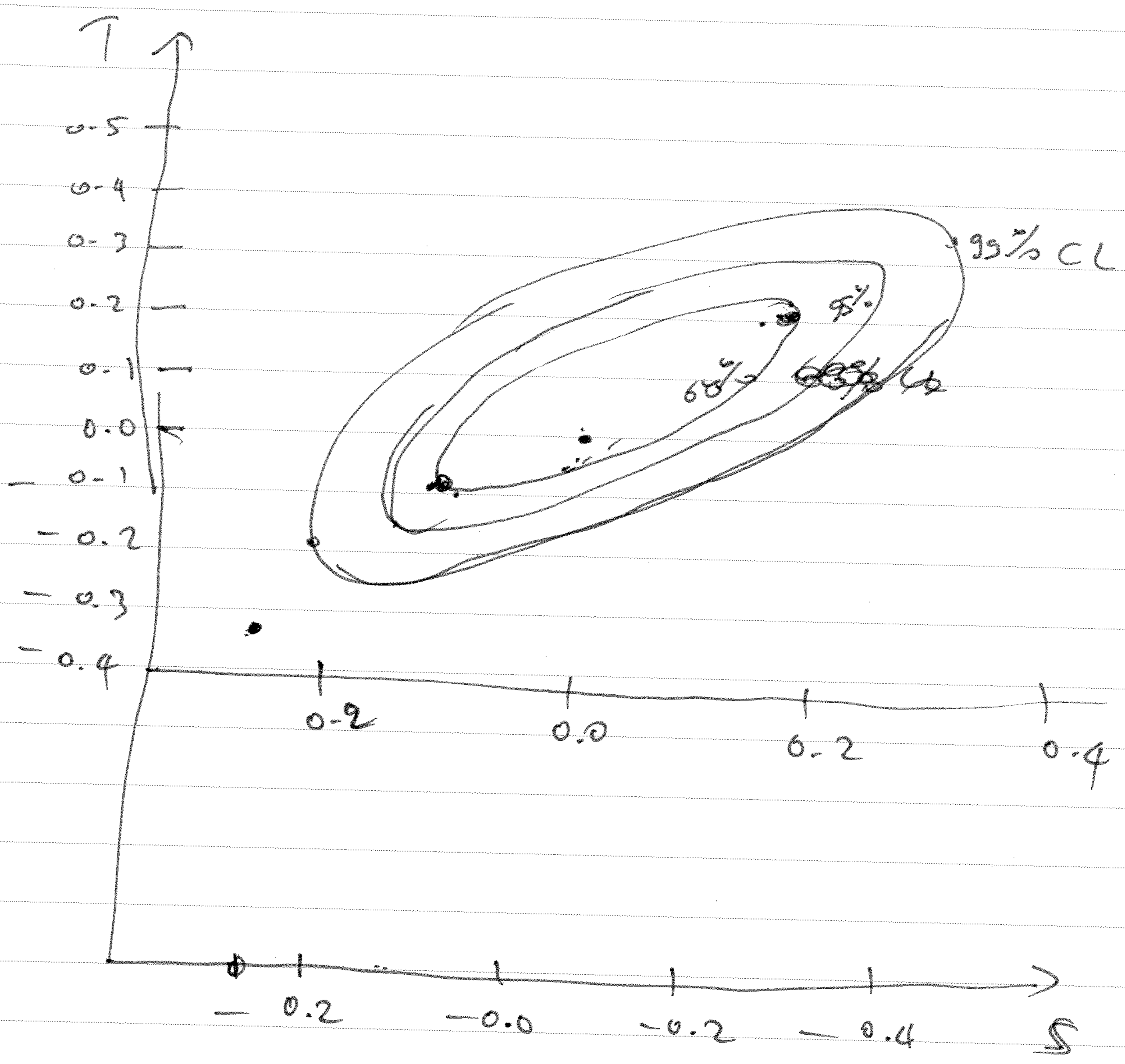
suppressed by new physics $\frac{m_Z^2}{m_N^2}$

In the SM,

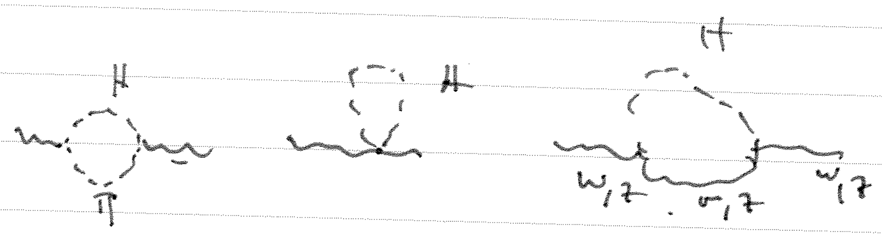
$S = 0.01 \pm 0.10$

$T = 0.03 \pm 0.11$

$U = 0.06 \pm 0.10$


T
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
95% CL
68%
0.2
0.0
0.2
0.4
- 0.2
-0.0
-0.2
- 0.4
S

Let us now look at the contribution of a Higgs boson:



$$S \approx \frac{1}{12\pi} \log\left(\frac{m_H^2}{m_{H,ref}^2}\right)$$

$$T \approx -\frac{3}{16\pi c^2} \log\left(\frac{m_H^2}{m_{H,ref}^2}\right)$$

$$U \approx 0.$$

~~Contribution of top quark.~~

or