ETH
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# The Physics of Electroweak symmetry breaking
# Exercise sheet 3

HS 2011

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### Exercise 3.1 Symmetry generators

Consider a scalar field theory and suppose we have a symmetry $S$ of the classical action (for simplicity, let's assume it is also a symmetry of the Lagrangian) and the path integral measure, that acts linearly on the fields,

$$\phi_n(x) \mapsto \phi_n(x) + \epsilon\delta\phi_n(x) = \phi_n(x) + i\epsilon\, t_{nm}\phi_m(x)\,.$$

Show that the *Noether charge* $Q$ associated to this symmetry generates the symmetry transformations, i.e.

$$[Q, \phi_n(x)] = -t_{nm}\phi_m(x)$$

*Hints:* $Q = \int \mathrm{d}^3x\, J^0(\vec{x}, 0)$, with $J^\mu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_m(x))}\delta\phi_m(x)$. Use the canonical commutation relations for the fields and their conjugated momenta.

### Exercise 3.2 Current translation

Prove eq. (19.2.45) in the book of Weinberg that reads

$$\langle\beta|J^\mu(x)|\alpha\rangle = e^{iq\cdot x}\,\langle\beta|J^\mu(0)|\alpha\rangle\ , \quad \text{with} \quad q^\mu := p^\mu_\alpha - p^\mu_\beta\,.$$

### Exercise 3.3 Example of Goldstone's theorem

Consider the following Lagrangian for $N$ real scalar fields,

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\Phi)^\mathsf{T}(\partial^\mu\Phi) - \frac{1}{2}m^2\Phi^\mathsf{T}\Phi - \frac{\lambda}{4}\left(\Phi^\mathsf{T}\Phi\right)^2\ ,$$

where $\Phi^\mathsf{T} = (\phi_1, \ldots, \phi_N)$. Convince yourself that it has a global $O(N)$ symmetry.

a) Now, assume that $m^2 < 0$. Find the vacua by finding the minima of the effective potential $V(\Phi)$ at tree level (i.e. just the classical potential). You should find that a vacuum $\Phi_0$ satisfies

$$\Phi_0^\mathsf{T}\Phi_0 = -\frac{m^2}{\lambda}\,.$$

b) Show that the mass matrix $M$ defined by

$$M_{nm} = \left.\frac{\partial^2 V(\Phi)}{\partial\phi_n\partial\phi_m}\right|_{\Phi=\Phi_0}$$

has only one eigenvalue different from zero, i.e. there are $(N-1)$ massless particles in the spectrum. Why exactly $(N-1)$?

*Hints:* Find an explicit eigenvector with an eigenvalue different from zero. Then use the fact that eigenvectors of symmetric matrices are orthogonal.