Statistical Physics
Exercise Sheet 8

Exercise 8.1 Independent Dimers in a Magnetic Field

We consider a system of N independent dimers of two spins, $s = 1/2$, described by the Hamiltonian

$$\mathcal{H}_0 = J \sum_i (\vec{S}_{i,1} \cdot \vec{S}_{i,2}) \tag{1}$$

where i numbers the dimers and the second index $m = 1, 2$ denotes their magnetic site. For simplicity, we use $\hbar = 1$.

a) What are the eigenstates and the eigenenergies of a single dimer? Consider the macroscopic system and determine the Helmholtz free energy, entropy, internal energy and specific heat as a function of temperature and N. Discuss the limit $T \to 0$ and $T \to \infty$ for both signs of J.

b) We now apply a magnetic field in z direction leading to an additional term in the Hamiltonian,

$$\mathcal{H}' = -g\mu_B H \sum_{i,m} S^z_{i,m}. \tag{2}$$

How do the eigenenergies change? Sketch the energies with respect to the applied field H and determine the ground state. Discuss in this context the entropy per dimer in the limit $T \to 0$.

c) Calculate the magnetization m and the magnetic susceptibility χ and discuss their dependence on H for different temperatures.

Exercise 8.2 Spin Susceptibility of the Free Electron Gas

We consider a Fermi-gas with a Zeeman-coupling g to an external magnetic field B,

$$\mathcal{H}_B = \frac{g}{2} B (N_+ - N_-) \, , \tag{3}$$

where N_+ and N_- are the occupation numbers of the two spin species.

1. Derive a representation of the grand-canonical potential Ω similar to the one in the lecture notes in the presence of this magnetic field. Show that it can be written as a sum $\Omega = \sum_\alpha \Omega_\alpha$ over the spin species $\alpha = \{+, -\}$ with a spin-dependent fugacity z_α. Find an expression for the spin-susceptibility in terms of $f_{n/2}(z_\alpha)$ (for some n) by acting with an appropriate differential operator on Ω.

2. Use the high-temperature expansion of $f_{n/2}$ to calculate the corresponding limiting behavior of the spin-susceptibility in the limit $B \to 0$.

3. Use the low-temperature expansion to do the same for $k_B T \ll \mu$.

Office Hours: Friday, 12.11, 15:00–17:00 at HIT K 33.3.