Exercise 1) Tensor Product Representations

The representation of $SU(2)$ on $\mathbb{C}^2$ is given by

$$\begin{aligned} V_1 : SU(2) &\to SU(2) \\ g &\mapsto g \ , \end{aligned}$$

the *defining* representation. How does the n-fold tensor product representation $V_1^{\otimes n}$ act on $(\mathbb{C}^2)^{\otimes n}$?

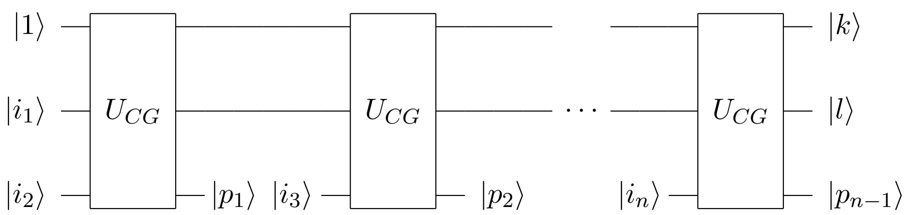
What is the corresponding representation of $su(2)$ on $(\mathbb{C}^2)^{\otimes n}$?

Exercise 2) Schur Transform

As we have seen in the lecture, we have

$$V_1^{\otimes n} \cong \bigoplus_k V_k \otimes \mathbb{C}^{m_k^n} \ .$$

The corresponding circuit is given by



If we take a computational basis state $|i_1 i_2 \cdots i_n\rangle$ as an input and then perform a measurement on the output register l, what is the measurement result?

Let $n = 2$ and imagine that the output of the Schur transform is $|k,l\rangle = |0,0\rangle$. What was the input? What was the input for the output $|k,l\rangle = |2,0\rangle, |2,1\rangle, |2,2\rangle$ resp. ?

Let $n = 3$ and imagine that the output of the Schur transform is $|k,l\rangle = |3,3\rangle$. What was the input?

Exercise 3) Asymptotic Behavior of m_k^n

We have seen in the lecture that the coefficients m_k^n are determined by $m_k^n = \binom{n}{\frac{n-k}{2}} \cdot (\frac{2k+2}{n+k+2})$ for $n \mod 2 = k \mod 2$ and vanish otherwise. In this exercise we calculate with what exponent m_k^n grows asymptotically for k linear in n.

Use Stirling's approximation $\lim_{n\to\infty}(\frac{n!}{\sqrt{2\pi n}(\frac{n}{e})^n}) = 1$ to show that

$$\lim_{n\to\infty} \frac{1}{n} \log(m_k^n) = h\left(\frac{1-c}{2}\right) \ ,$$

where $k = c \cdot n$ and $h(x) = -x \log x - (1-x)\log(1-x)$ is the binary entropy function. All logarithms are to base two and e is Euler's number.