Quantum Field Theory III

HS 10, Exercise sheet 7

Due date: 10.11.2010

Exercise 1:
In the lecture you saw the gauge superfield in Wess-Zumino gauge[1]

$$V(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}v_\mu + i(\theta\theta)\bar{\theta}\bar{\lambda} - i(\bar{\theta}\bar{\theta})\theta\lambda + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D.$$

This gauge breaks supersymmetry. This means a supersymmetry variation of a gauge superfield in the Wess-Zumino gauge will not remain in the Wess-Zumino gauge. In order to get it back to this gauge it will be necessary to perform a compensating gauge transformation.
In this exercise you should first compute the supersymmetry transformation of an abelian gauge superfield in Wess-Zumino gauge and the calculate the compensating gauge transformation to go back to the Wess-Zumino gauge.

Exercise 2: PhD exercise
The goal of this PhD exercise is to investigate the behaviour of simple supersymmetric models under transformations of the conformal algebra.
The conformal algebra of Minkowski space contains the Poincare algebra as a subalgebra, and in addition it has five other generators: the **dilation** D and the **special conformal transformations** K_μ. These give the following commutation relations

$$\begin{aligned} [P_\mu, D] &= P_\mu, \\ [K_\mu, D] &= -K_\mu, \\ [P_\mu, K_\nu] &= 2\eta_{\mu\nu}D - 2M_{\mu\nu}, \\ [M_{\mu\nu}, K_\rho] &= \eta_{\nu\rho}K_\mu - \eta_{\mu\rho}K_\nu. \end{aligned} \tag{1}$$

a) How do the generators of the conformal algebra (P_μ, $M_{\mu\nu}$, D, K_μ) have to act on a scalar field S, a pseudoscalar field P and a spinor ψ, such that the conformal algebra is fullfilled?

b) Prove that the action of the massless Wess-Zumino Lagrangian

$$\mathcal{L}_{WZ} = -\frac{1}{2}(\partial S)^2 - \frac{1}{2}(\partial P)^2 - \frac{1}{2}\bar{\psi}\partial_\mu\gamma^\mu\psi - \lambda\bar{\psi}(S - P\gamma_5)\psi - \frac{1}{2}\lambda^2(S^2+P^2)^2$$

is conformal invariant.

c) We know that the Wess-Zumino Lagrangian is invariant under both supersymmetry and conformal transformations. The commutator of an infinitesimal supersymmetry and an infinitesimal special conformal transformation is, by definition, a **conformal supersymmetry**. These are generated by a spinorial generator S_a, defined by

$$[K_\mu, Q_a] = (\gamma_\mu)_a{}^b S_b.$$

The infinitesimal conformal supersymmetry is $\delta_\zeta\phi = \bar{\zeta}S\cdot\phi$, where ζ is an anticommuting Majorana spinor. How does δ_ζ act on S, P and ψ?

[1] Here we consider only an abelian gauge field.

d) The last step is to show that the superconformal algebra on the fields S, P and ψ closes on-shell. For this we need to introduce the R-symmetry generator R. How does this generator act on S, P and ψ? What additional commutation relations do you get for the superconformal algebra, consisting of the generators P_μ, $M_{\mu\nu}$, D, K_μ, Q_a, S_a and R?

e) How can one interpret the dilation D and the special conformal transformations K_μ physically? How can you view them pictorially?

f) Are there other Lagrangians that are invariant under superconformal transformations? *Hint:* What happens when you add gauge fields? Can a Lagrangian with masses or massive couplings be invariant under dilations?