Quantum Field Theory III

HS 10, Exercise sheet 5

Due date: 27.10.2010

Exercise 1:
The most general generic component expansion of the superfield is

$$\Phi(x,\theta,\bar\theta) = f(x)+\theta\phi(x)+\bar\theta\bar\chi(x)+\theta\theta m(x)+\bar\theta\bar\theta n(x)+\theta\sigma^\mu\bar\theta v_\mu(x)+(\theta\theta)\bar\theta\bar\lambda(x)+(\bar\theta\bar\theta)\theta\psi(x)+(\theta\theta)(\bar\theta\bar\theta)d(x).$$

Calculate $\Phi_1(x,\theta,\bar\theta)\Phi_2(x,\theta,\bar\theta)$. Use spinor identities to bring it to the same form as the component expanded superfield.

Exercise 2:
Prove the following superspace identities:

a) $\theta^\alpha\theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta$

b) $\theta_\alpha\theta_\beta = -\frac{1}{2}\epsilon_{\alpha\beta}\theta\theta$

c) $(\theta\psi)(\theta\chi) = -\frac{1}{2}(\theta\theta)(\psi\chi)$

d) $(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$

Exercise 3:
The covariant derivatives are

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu,$$
$$\bar D_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu.$$

Show that $\bar D^2\mathcal{F}(x,\theta,\bar\theta)$ is a chiral superfield, where $\mathcal{F}(x,\theta,\bar\theta)$ is a general superspace-function and $\bar D^2 = \bar D_{\dot\alpha}\bar D^{\dot\alpha}$. Aquivalently show that $D^2\mathcal{F}(x,\theta,\bar\theta)$ is antichiral.
Hint: What are $\bar D_{\dot\alpha}\bar D^2$ and $D_\alpha D^2$ respectively?

Exercise 4:
A chiral superfield is the most general function of the bosonic coordinate $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$ and θ^α, which can be parameterized as

$$\Phi(y,\theta) = z(y) + \sqrt{2}\theta\psi(y) - \theta\theta F(y).$$

It is obvious that the product of two (anti)chiral superfields is again a(n) (anti)chiral superfield. To construct the kinetic term of the Lagrangian we need the D-term ($\theta\theta\bar\theta\bar\theta$-term) of the product of a chiral superfield and an antichiral superfield. Calculate

$$\bar\Phi\Phi|_{\theta\theta\bar\theta\bar\theta}.$$

Hint: To do this you need to taylor expand the bosonic coordinate, i.e. you need to show that

$$\Phi(y,\theta) = z(x) + \sqrt{2}\theta\psi(x) - \theta\theta F(x) + i\theta\sigma^\mu\bar\theta\partial_\mu z(x) - \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi(x)\sigma^\mu\bar\theta - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box z(x).$$