Quantum Field Theory III

HS 10, Exercise sheet 2

Due date: 06.10.2010

Exercise 1:
In the lecture we derived the commutation relation $\left\{Q^I_\alpha, \bar{Q}^J_{\dot{\alpha}}\right\} = 2\delta^{IJ}(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu$ and the definition for the central charges $\left\{Q^I_\alpha, Q^J_\beta\right\} = \epsilon_{\alpha\beta}Z^{IJ}$. By using these relations plus the Jacobi identity show the following commutation relations

a) $\left[\bar{Q}^K_{\dot{\gamma}}, Z^{IJ}\right] = 0$,

b) $\left[Q^K_\gamma, (Z^{IJ})^\star\right] = 0$.

Exercise 2:
In the lecture we showed the commutation relations $\left[Z^{IJ}, Q^K_\alpha\right] = 0$ and $\left[(Z^{IJ})^\star, \bar{Q}^K_{\dot{\alpha}}\right] = 0$. By using these relations plus the Jacobi identity show the following commutation relations

a) $\left[Z^{IJ}, Z^{KL}\right] = 0$,

b) $\left[(Z^{IJ})^\star, (Z^{KL})^\star\right] = 0$,

c) $\left[Z^{IJ}, (Z^{KL})^\star\right] = 0$.

Exercise 3:
The internal symmetry group fullfills the relation $[B_a, B_b] = if^c_{ab}B_c$, where f^c_{ab} are the structure constants. Moreover the internal symmetries do not commute with the fermionic generators, but obey $\left[Q^I_\alpha, B_l\right] = S^I_{lJ}Q^J_\alpha$ and $\left[\bar{Q}^I_{\dot{\alpha}}, B_l\right] = -\bar{Q}^J_{\dot{\alpha}}S_{lJ}{}^I$. Using the Jacobi identity of B_a, B_b and Q^I_α prove that

$$[S_a, S_b] = if^c_{ab}S_c.$$

Exercise 4:
Show that P^2 is a Casimir of the supersymmetry algebra, i.e. show that it commutes with all operators of the supersymmetry algebra.

Exercise 5:
Show that W^2 does not commute with the fermionic generators, i.e. $\left[W^2, Q^I_\alpha\right] \neq 0$ and thus W^2 is not a Casimir of the supersymmetry algebra.