Quantum Field Theory III

HS 10, Exercise sheet 10

Due date: 01.12.2010

Exercise 1:

a) Compute the supercurrent for the massless Wess-Zumino model, with Lagrangian

$$\mathcal{L} = (\partial_\mu z)^\dagger (\partial^\mu z) - i\psi\sigma^\mu \partial_\mu \bar{\psi} + F^\dagger F.$$

To do this you first compute the usual Noether current, which is given by

$$\sum_l \delta\chi^l \frac{\partial_L \mathcal{L}}{\partial(\partial_\mu \chi^l)} \equiv -(\epsilon^\alpha N^\mu_\alpha + \bar{N}^\mu_{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}}),$$

where $\delta\chi^l$ are the supersymmetry transformations of the various fields. For a chiral multiplet we use

$$\begin{aligned} \delta z &= \sqrt{2}\epsilon\psi, \\ \delta\psi_\alpha &= -\sqrt{2}\epsilon_\alpha F + \sqrt{2} i \sigma^\mu_{\alpha\dot{\beta}} \bar{\epsilon}^{\dot{\beta}} \partial_\mu z, \\ \delta F &= \sqrt{2} i \partial_\mu \psi \sigma^\mu \bar{\epsilon}. \end{aligned}$$

We know that the action is invariant under supersymmetry, but the variation of the Lagrangian is always a total derivative

$$\delta\mathcal{L} = \epsilon^\alpha \partial_\mu K^\mu_\alpha + \partial_\mu \bar{K}^\mu_{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}}$$

It is easy to show that the divercence of the usual Noether current is also related to the variation of the Lagrangian

$$\epsilon^\alpha \partial_\mu N^\mu_\alpha + \partial_\mu \bar{N}^\mu_{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}} = -\delta\mathcal{L}.$$

Putting everything togeter we obtain a conserved quantity, which we call the supersymmetry current

$$S^\mu \equiv N^\mu + K^\mu, \quad \text{with} \quad \partial_\mu S^\mu = 0.$$

b) Compute the change in the supersymmetry current when you add to the massless Wess-Zumino Lagrangian the Lagrangian with the superpotential

$$\mathcal{L}_W = \left[\left(\frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 \right)_{\text{F-term}} + h.c. \right].$$