Particle Physics Phenomenology I

HS 10, Series 5

Due date: 29.10.2010, 1 pm

Exercise 1

(i) Show that the chirality is not a good quantum number for a massive fermion by checking $[H, \gamma_5]$.

(ii) Show that helicity is conserved although it depends on the choice of the coordinate system.

Exercise 2 Show that

$$\gamma^5 u = \begin{pmatrix} \vec{\sigma}\cdot\vec{p} & 0 \\ 0 & \vec{\sigma}\cdot\vec{p} \end{pmatrix} u \tag{1}$$

holds in the ultrarelativistic limit, i.e. that the chirality operator γ^5 coincides with the helicity operator.

Exercise 3 The Lorentz gauge condition $k.\epsilon_\lambda(k) = 0$ is not sufficient to fix the two independent components of the polarisation vectors $\epsilon^\mu_\lambda(k)$. The "axial gauge" imposes another condition, namely $n.\epsilon_\lambda(k) = 0$ where n is some arbitrary 4-vector which satisfies $n.k \neq 0$. Further more the polarisation vectors must satisfy the orthonormality condition

$$\epsilon_{\lambda_1}(k).\epsilon^*_{\lambda_2}(k) = -\delta_{\lambda_1\lambda_2}.$$

Show that the polarisation (sum) tensor of a massless spin 1 particle (e.g. photons, gluons) is given by

$$P^{\mu\nu} \equiv \sum_\lambda \epsilon^\mu_\lambda(k)\epsilon^{*\nu}_\lambda(k) = -g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n.k} - \frac{n^2 k^\mu k^\nu}{(n.k)^2}. \tag{2}$$

Hint: Write down all possible rank 2 tensors upon which $P^{\mu\nu}$ may depend. Then use the 5 stated conditions to determine their coefficients.

Remark: In QED it is sufficient to take

$$P^{\mu\nu} \equiv \sum_\lambda \epsilon^\mu_\lambda(k)\epsilon^\nu_\lambda(k) = -g^{\mu\nu} \tag{3}$$

This is due to the Takahashi-Ward identity which demands that $k_\mu M^\mu = 0$ for some n-point greensfunction M^μ (matrix element) which has an external photon of momentum k_μ.