Particle Physics Phenomenology I

HS 10, Series 11

Due date: 17.12.2010, 1 pm

Exercise 1 Prove the Fierz Identity for the SU(N) generators

$$\sum_a (T^a)_{ij}(T^a)_{kl} = T_F(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}) \tag{1}$$

Hint: Use completeness, i.e. $M_{ij} = \sum_a m^a (T^a)_{ij} + m^0 \delta_{ij}$ where M is an arbitrary $N \times N$ matrix.

Exercise 2 Consider the kinematics for the process $\gamma^* \to q\bar{q}g$.
Define the following Lorentz invariant quantities

$$x_i = \frac{2p_i.p_{CM}}{Q^2}, \quad i = q, \bar{q}, g, \quad p_{CM} = (Q, 0, 0, 0). \tag{2}$$

(i) Show that

$$x_q + x_{\bar{q}} + x_g = 2. \tag{3}$$

What is the range of any of the x_i?

(ii) Show that if any $x_i \to 0$ this means that $E_i \to 0$ (particle i becomes soft). Further more show that only one particle can become soft at the time.

(iii) Show that if any $x_i \to 1$ the remaining two particles, unless one of them is soft, are collinear to each other and hence anticollinear to particle i.

(iv) If we define the Lorentz invariants

$$y_{ij} = \frac{2p_i.p_j}{Q^2}, \quad i, j = q, \bar{q}, g, \quad i \neq j \tag{4}$$

show that

$$y_{ij} = 1 - |\epsilon_{ijk}| x_k \tag{5}$$

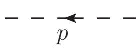
where $|\epsilon_{ijk}|$ equals 1 if $i \neq j \neq k$ and 0 otherwise.

(iv) The JADE jet algorithm only accepts events which fulfill $y_{ij} > y_{\mathbf{min}} > 0$. What implications does this have on the Daltiz plot (make a sketch)? Which physical configurations are being rejected?

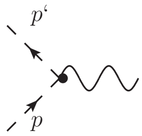
Exercise 3 This problem concerns the theory of a complex scalar field ϕ interacting with the electromagnetic field A^μ. The Lagrangian is given by

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{6}$$

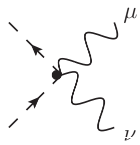
where $D^\mu = \partial^\mu + ieA^\mu$. The Feynman rules for this theory are



$= \frac{i}{p^2-m^2+i\epsilon}$



$= -ie(p+p')^\mu$



$= 2ie^2 g^{\mu\nu}$

as well as the usual Feynman rule for the photon propagator.

(i) Show that the Lagrangian is invariant under local gauge transformations, i.e. $\phi(x) \to e^{i\theta(x)}\phi(x)$ and $A^\mu(x) \to A^\mu(x) - \frac{1}{e}\partial^\mu\theta(x)$.

(ii) Ignoring particle masses, compute, to lowest order, the spin averaged differential cross section for $e^+e^- \to \phi\phi^*$. Find the asymptotic angular dependence and compare your results to the corresponding formulae for $e^+e^- \to \mu^+\mu^-$.