

Particle Physics Phenomenology I

HS 10, Series 10

Due date: 10.12.2010, 1 pm

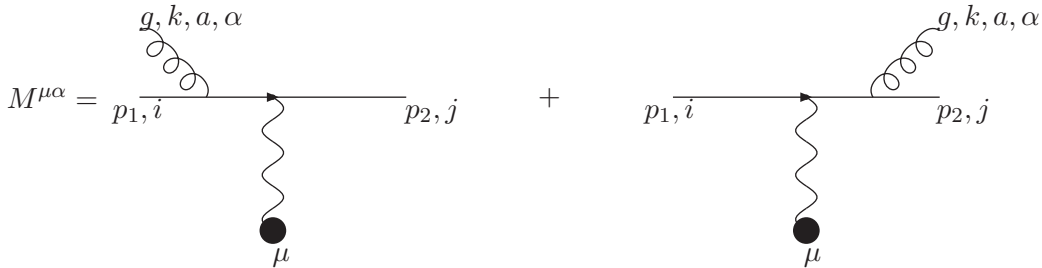
Exercise 1 The generators $T^a(R)$ of any representation R obey the commutation rules

$$[T^a(R), T^b(R)] = if^{abc} T^c(R). \quad (1)$$

Show that the generators T^a of the adjoint representation, given by $(T^a)_{bc} = -if_{abc}$, satisfy this commutation relations.

Hint: Use the Jacobi Identity ($[[T^a, T^b], T^c] + [[T^b, T^c], T^a] + [[T^c, T^a], T^b] = 0$).

Exercise 2 Consider soft radiative QCD corrections to a quark scattering off an electromagnetic current.



a) Show that (in the soft limit) the Matrix element becomes

$$M_{ij}^{\mu\alpha a} = (-ie\bar{u}_2\gamma^\mu u_1) J_{ij}^{\alpha a} (1 + O(k))$$

where

$$J_{ij}^{\alpha a} = -g(T^a)_{ji} \left[\frac{p_2^\alpha}{p_2 \cdot k} - \frac{p_1^\alpha}{p_1 \cdot k} \right].$$

Hence prove that $J_{ij}^{\alpha a} k_\alpha = 0$ to show that the current J is conserved.

b) Compute the square of the matrixelement, i.e.

$$|M^{\mu\nu}|^2 = \sum_\lambda \epsilon_\alpha(\lambda) (\epsilon^*)_\beta(\lambda) M_{ij}^{\mu\alpha a} (M^*)_{ij}^{\nu\beta a}.$$

and show that the Matrix element square factorises into a soft (eikonal) part times the squared amplitude of the underlying leading order process ($|M_0^{\mu\nu}|^2$) as follows

$$|M^{\mu\nu}|^2 = -g^2 C_F \left(\frac{p_1^2}{p_1 \cdot k} - \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} + \frac{p_2^2}{p_2 \cdot k} \right) |M_0^{\mu\nu}|^2.$$

Exercise 3 Using the fact that a QCD amplitude can be written as the product of a purely kinematical (Lorentz invariant) part times a colour part, calculate the colour factors that appear in the following squared matrix elements at leading order: $|\mathcal{M}(qq' \rightarrow qq')|^2$, $|\mathcal{M}(q\bar{q} \rightarrow q'\bar{q}')|^2$, $|\mathcal{M}(qg \rightarrow qg)|^2$, $|\mathcal{M}(q\bar{q} \rightarrow gg)|^2$.