Classical Mechanics
Exercise 5

HS 09
Prof. B. Moore

Exercise 5.1 Moving guiding curve : rotating pendulum

Let us consider the frictionless motion of the material point of mass m in the gravitational field $\boldsymbol{g}$ constrained to move on a rotating circle. The circle of radius a is in a vertical plane and rotates with the constant angular velocity Ω around the vertical axis which passes through its center. Study the motion of the system and discuss the different regimes.

Hints :

- Start by writing the vectorial differential equation of motion (Newton's equation) expressed in a frame of reference rotating with the circle.
- Integrate it.
- Using the dimensionless variable $\tau = \omega t$ and the dimensionless number $n = \Omega/\omega$ where $\omega = \sqrt{\frac{g}{a}}$, show that the equation of motion can be written as

  $$\frac{\mathring{\theta}^2}{2} - (\frac{n^2}{2}\sin^2\theta + \cos\theta) = c$$

  where θ is the angle in the rotating plane took from the vertical symmetry axis of the circle, where $^\circ$ is the derivative w.r.t τ and c is a constant fixed by initial conditions.
- Study the motion of the system.
  Study the function (give its physical interpretation)

  $$\mathcal{V}(\theta) = -\frac{n^2}{2}\sin^2\theta - \cos\theta,$$

  for each cases :

  - $n < 1$,
  - $n > 1$,
  - $n = 1$,

  and describe the different regimes as a function of c (give the physical interpretation of c).
  Plot the corresponding trajectories in the phase diagram in terms of the couples $(\theta, \mathring{\theta})$ (use for example the function ContourPlot[ ] in Mathematica).

Exercise 5.2 Variable mass system

A space shuttle starting at rest goes up vertically in the uniform gravitational field $\boldsymbol{g}$. The propulsion is due to the backward ejection of the combustion gas at the constant relative velocity w. The total mass of the shuttle is the sum of the mass of the fuel m_f and the mass of the payload m_l. Show that the velocity reached by the space shuttle at the end of the combustion does not depend on the specific form of the ejection law as a function of time $m(t)$ and only depends on the total duration of the combustion t_c, on g, on w and on the ratio $\dfrac{m_f}{m_l}$. We neglect the rotation of the Earth.