Statistical Physics
Exercise Sheet 2

Exercise 2.1 Thermodynamics of a magnetic system, part II

Consider a magnetic system whose differential work is given by $\delta W = -HdM$. The thermodynamics of such a system is governed by the thermal and caloric equations of state: $U = U(T, M)$, $H = H(T, M)$.

(a) Show that

$$C_M - C_H = \left(H - \left.\frac{\partial U}{\partial M}\right|_T \right) \left.\frac{\partial M}{\partial T}\right|_H \quad (1)$$

$$= T \left.\frac{\partial H}{\partial T}\right|_M \left.\frac{\partial M}{\partial T}\right|_H . \quad (2)$$

Hint: Use a Maxwell relation (for which variables?) to show the second equation.

(b) Show that the internal energy U depends only on the temperature T if and only if the thermal equation of state is of the form $H = Tf(M)$. Compute $C_M - C_H$ in this case.

(c) Show that C_M depends only on T if and only if $H = g(M) + Tf(M)$.

Hint: Show first that

$$\left.\frac{\partial C_M}{\partial M}\right|_T = -T \left.\frac{\partial^2 H}{\partial T^2}\right|_M . \quad (3)$$

Exercise 2.2 Curie-Weiss paramagnet

A paramagnetic substance obeys the Curie-Weiss law, i.e.,

$$M = \frac{C}{T - \theta} H, \quad (4)$$

with constants C and θ. Show, that the internal energy U and the entropy S are given by the following expressions,

$$U = \int^T C_M dT - \frac{\theta}{2C} M^2 + \text{const.}, \quad (5)$$

$$S = \int^T C_M \frac{dT}{T} - \frac{M^2}{2C} + \text{const.} \quad (6)$$