Exercises for "Phenomenology of Particle Physics I"

Prof. Dr. A. Gehrmann
M. Ritzmann

sheet 6

handed out: 28.10.2008
handed in: 4.11.2008
returned: 11.11.2008

http://www.itp.phys.ethz.ch/education/lectures_hs08/PPPI

Exercise 16

This exercise treats the photon polarisations. Let k denote the photon momentum and p an arbitrary lightlike ($p^2 = 0$) momentum with $k \cdot p \neq 0$. We define the spinors of a massless particle with positive/negative helicity and momentum q as $u_R(q)$ and $u_L(q)$. We have

$$\frac{(1+\gamma^5)}{2}u_R = u_R \quad \frac{(1-\gamma^5)}{2}u_R = 0 \quad \frac{(1+\gamma^5)}{2}u_L = 0 \quad \frac{(1-\gamma^5)}{2}u_L = u_L.$$

We define the polarisation vectors as

$$\epsilon_+^\mu(k) = \frac{1}{\sqrt{4p\cdot k}}\bar{u}_R(p)\gamma^\mu u_R(k) \qquad \epsilon_-^\mu(k) = \frac{1}{\sqrt{4p\cdot k}}\bar{u}_L(p)\gamma^\mu u_L(k).$$

(i) Show that these vectors are in fact polarisation vectors, i.e.

  (a) $\epsilon_\pm^\mu(k)k_\mu = 0$,
  (b) $\epsilon_+(k)\epsilon_-^*(k) = 0$.

(ii) Use the identities

  (a) $u_L(p)\bar{u}_L(p) + u_R(p)\bar{u}_R(p) = \not{p}$
  (b) $x^\dagger A y = Tr\left(y x^\dagger A\right)$
  (c) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4\left[g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}\right]$

to show that

$$\epsilon_+^\mu\epsilon_+^{*\nu} + \epsilon_-^\mu\epsilon_-^{*\nu} = -g^{\mu\nu} + \frac{k^\mu p^\nu + k^\nu p^\mu}{p\cdot k}$$

holds.

– please turn over –

Exercise 17

We choose the three polarisation states of a massive spin 1 particle with mass m and four-momentum $(E, 0, 0, |\vec{p}|)$ as

$$\begin{aligned}\epsilon^{(\lambda=\pm)} &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \\ \epsilon^{(\lambda=0)} &= \frac{1}{m}(|\vec{p}|\,, 0, 0, E).\end{aligned}$$

Show that the completeness relation

$$\sum_{\lambda} \left(\epsilon_{\mu}^{(\lambda)}\right)^{*} \epsilon_{\nu}^{(\lambda)} = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{m^2}$$

holds for these three polarization vectors.