Exercises for "Phenomenology of Particle Physics I"

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sheet 5

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Exercise 12

In the lecture the solutions

$$u_\pm(p) = \sqrt{p^0+m}\begin{pmatrix} \chi_\pm \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{p^0+m}\,\chi_\pm \end{pmatrix}, \qquad v_\pm(p) = \sqrt{p^0+m}\begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{p^0+m}\,\chi_\mp \\ \chi_\mp \end{pmatrix}$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

of the Dirac equation (in natural units)

$$(\gamma^\mu p_\mu - m)u_\pm = 0$$
$$(\gamma^\mu p_\mu + m)v_\pm = 0$$

were presented. Verify that they are solutions of the Dirac equation.

Hint: Choose an appropriate coordinate system for p.

Exercise 13

Show the orthogonality of the solutions of the Dirac equation, means

$$\bar{u}^r(p)u^s(p) = 2m\delta^{rs} \quad \text{or} \quad (u^r(p))^\dagger\, u^s(p) = 2E(\vec{p})\delta^{rs}$$
$$\bar{v}^r(p)v^s(p) = -2m\delta^{rs} \quad \text{or} \quad (v^r(p))^\dagger\, v^s(p) = 2E(\vec{p})\delta^{rs}$$
$$\bar{u}^r(p)v^s(p) = \bar{v}^r(p)u^s(p) = 0.$$

– please turn over –

Exercise 14

Derive the spin sum formulae

$$\sum_{s=+,-} u^s(p)\bar{u}^s(p) = \not{p} + m \tag{1}$$

$$\sum_{r=+,-} v^r(p)\bar{v}^r(p) = \not{p} - m. \tag{2}$$

Exercise 15

(i) Show that the chirality is not a good quantum number for a massive fermion by checking $[H, \gamma_5]$.

(ii) Show that helicity is conserved although it depends on the choice of the coordinate system.