## Exercises for "Phenomenology of Particle Physics I"

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http://www.itp.phys.ethz.ch/education/lectures_hs08/PPPI returned: 4.11.2008

## Exercise 12

In the lecture the solutions

$$
\begin{array}{cc}
u_{ \pm}(p)=\sqrt{p^{0}+m}\binom{\chi_{ \pm}}{\frac{\vec{\sigma} \cdot \vec{p}}{p^{0}+m} \chi_{ \pm}}, & v_{ \pm}(p)=\sqrt{p^{0}+m}\binom{\frac{\vec{\sigma} \cdot \vec{p}}{p^{0}+m} \chi_{\mp}}{\chi_{\mp}} \\
\chi_{+}=\binom{1}{0}, & \chi_{-}=\binom{0}{1}
\end{array}
$$

of the Dirac equation (in natural units)

$$
\begin{aligned}
\left(\gamma^{\mu} p_{\mu}-m\right) u_{ \pm} & =0 \\
\left(\gamma^{\mu} p_{\mu}+m\right) v_{ \pm} & =0
\end{aligned}
$$

were presented. Verify that they are solutions of the Dirac equation.

Hint: Choose an appropriate coordinate system for $p$.

## Exercise 13

Show the orthogonality of the solutions of the Dirac equation, means

$$
\begin{gathered}
\bar{u}^{r}(p) u^{s}(p)=2 m \delta^{r s} \quad \text { or } \quad\left(u^{r}(p)\right)^{\dagger} u^{s}(p)=2 E(\vec{p}) \delta^{r s} \\
\bar{v}^{r}(p) v^{s}(p)=-2 m \delta^{r s} \quad \text { or } \quad\left(v^{r}(p)\right)^{\dagger} v^{s}(p)=2 E(\vec{p}) \delta^{r s} \\
\bar{u}^{r}(p) v^{s}(p)=\bar{v}^{r}(p) u^{s}(p)=0 .
\end{gathered}
$$

## Exercise 14

Derive the spin sum formulae

$$
\begin{align*}
& \sum_{s=+,-} u^{s}(p) \bar{u}^{s}(p)=\not p+m  \tag{1}\\
& \sum_{r=+,-} v^{r}(p) \bar{v}^{r}(p)=\not p-m . \tag{2}
\end{align*}
$$

## Exercise 15

(i) Show that the chirality is not a good quantum number for a massive fermion by checking $\left[H, \gamma_{5}\right]$.
(ii) Show that helicity is conserved although it depends on the choice of the coordinate system.

