Institute for Theoretical Physics, ETH Zürich HS 2008

Exercises for "Phenomenology of Particle Physics I"

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sheet 4

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http://www.itp.phys.ethz.ch/education/lectures_hs08/PPPI

Exercise 8

(factors of c corrected)

Use the Euler-Lagrange equation on the Lagrangian density of the real Klein-Gordon field

$$\mathcal{L} = \frac{1}{2}\hbar^2(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}c^2m^2\phi^2 \tag{1}$$

to derive the real Klein-Gordon equation

$$\left(\partial_\mu\partial^\mu + \frac{m^2c^2}{\hbar^2}\right)\phi = 0. \tag{2}$$

Then do the same for the Lagrangian density of the complex Klein-Gordon field

$$\mathcal{L} = \hbar^2(\partial_\mu\phi)(\partial^\mu\phi^*) - c^2m^2\phi\phi^* \tag{3}$$

to derive the Klein Gordon equation for ϕ as well as for ϕ^*.

Exercise 9

Show that the Lagrangian density

$$\mathcal{L} = \bar{\psi}(ic\hbar\gamma^\mu\partial_\mu - mc^2)\psi \tag{4}$$

leads to the Dirac equations for ψ and $\bar{\psi}$ given by

$$\left(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar}\right)\psi = 0, \quad i\partial_\nu\bar{\psi}\gamma^\nu + \frac{mc}{\hbar}\bar{\psi} = 0. \tag{5}$$

Exercise 10

Use the Dirac equations derived above to show

$$\partial_\mu j^\mu = 0 \qquad \text{(continuity equation)} \tag{6}$$

where j is defined by $j^\mu = \bar{\psi}\gamma^\mu\psi$.

– please turn over –

Exercise 11

(factors of c corrected)

Show that a solution of the Dirac Equation satisfies the Klein-Gordon equation for each of its spinor components by verifying

$$\left(\partial_\mu \partial^\mu + \frac{m^2}{\hbar^2}\right) = -\left(i\gamma^\nu \partial_\nu + \frac{mc}{\hbar}\right)\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right) \tag{7}$$