# General relativity, solution sheet 7.

*HS 08*

**1. Charged dust**

i) The equations of motion are (see (5.2), (4.11)):

$$(\rho_{\mathrm{m}} u^\mu)_{;\mu} = 0\,, \tag{1}$$

$$(\nabla_u u)^\mu = u^\nu u^\mu{}_{;\nu} = \frac{e}{mc} F^{\mu\nu} u_\nu\,, \tag{2}$$

where $\rho_{\mathrm{m}}(x) = n(x)m$ is the mass density. As the charge density $\rho_{\mathrm{el}}(x) = n(x)e = \rho_{\mathrm{m}}(x)e/m$ is a multiple of the mass density, $j^\mu{}_{;\mu} = 0$ is equivalent to (1).

ii) From the hint it follows that

$$T^{\mu\nu}_{\mathrm{em};\nu} = -\frac{1}{c} F^{\mu\nu} j_\nu\,.$$

With $T^{\mu\nu}_{\mathrm{d}} = \rho_{\mathrm{m}} u^\mu u^\nu$ we calculate

$$T_{\mathrm{d}}^{\mu\nu}{}_{;\nu} = (u^\mu(\rho_{\mathrm{m}} u^\nu))_{;\nu} \overset{(1)}{=} \rho_{\mathrm{m}} u^\mu{}_{;\nu} u^\nu \overset{(2)}{=} \frac{1}{c} F^{\mu\nu}(\rho_{\mathrm{el}} u)_\nu = -T^{\mu\nu}_{\mathrm{em};\nu}\,.$$

**2. Bound on the cosmological constant**

i) We write the field equations using the Ricci tensor (see (5.9)); the additional term on the right-hand side is

$$\Lambda(g^{\mu\nu} - \frac{1}{2}\delta^\alpha{}_\alpha g^{\mu\nu}) = -\Lambda g^{\mu\nu}\,.$$

The (00)-component of the field equations is in the Newtonian limit (see pgg. 39-40)

$$c^{-2}\Delta\varphi = \frac{1}{2}\kappa\rho c^2 - \Lambda \qquad ,\text{i.e.} \qquad \Delta\varphi = 4\pi G_0 \rho - \Lambda c^2\,.$$

ii) For a point mass $M$ at the origin, i.e. $\rho(x) = M\delta(x)$, the solution

$$\varphi(\vec{x}) = -\frac{G_0 M}{r}$$

needs the additional term $-1/6\Lambda c^2 r^2$, so that the gravitational acceleration is

$$-\nabla\varphi(\vec{x}) = -G_0 M \frac{\vec{x}}{r^3} + \frac{1}{3}\Lambda c^2 \vec{x}\,. \tag{3}$$

iii) The additional term in (3) is significant for large distances, but is negligible w.r.t. the Newton acceleration as long as $G_0 M r^{-2} \gg \Lambda c^2 r$, i.e. for

$$\Lambda \ll \frac{G_0 M}{c^2 r^3}\,.$$

For the solar system this implies $\Lambda \ll 6 \cdot 10^{-36}\mathrm{m}^{-2}$.