Exercise Sheet II

Hand in by 08.10.2008

Problem 1 [*Stretched string and a nonrelativistic limit.*]: Examine the action

$$S = -T_0 \int dt \int_0^{\sigma_1} d\sigma \left(\frac{ds}{d\sigma}\right) \sqrt{1 - \frac{v_\perp^2}{c^2}}$$

for a relativistic string with endpoints attached at $(0, \vec{0})$ and $(a, \vec{0})$. Consider the nonrelativistic approximation where $|\vec{v}_\perp| \ll c$ and the oscillations are small ($\frac{dy}{dx} \ll 1$). You may denote by $\vec{y}$ the collection of transverse coordinates $X^2, \ldots, X^d$ and write $\vec{y}(t, x)$, where x is the coordinate corresponding to X^1. Explain why the following relations hold:

$$ds^2 = dx^2 + d\vec{y} \cdot d\vec{y}, \quad \vec{v}_\perp \simeq \frac{\partial y}{\partial t}.$$

Show that the action reduces, up to an additive constant, to the *action* for a nonrelativistic string performing small transverse oscillations. What are the tension and the linear mass density of the resulting string? What is the additive constant?

Problem 2 [*Time evolution of a closed circular string.*]: At $t = 0$, a closed string forms a circle of radius R on the (x, y) plain and has zero velocity. The time development of this string can be studied using the action

$$S = -T_0 \int dt \int_0^{\sigma_1} d\sigma \left(\frac{ds}{d\sigma}\right) \sqrt{1 - \frac{v_\perp^2}{c^2}}.$$

The string will remain circular, but its radius will be a time-dependent function $R(t)$. Give the Lagrangian L as a function of $R(t)$ and its time derivative. Calculate the radius and velocity as functions of time. Sketch the spacetime surface traced by the string in a three-dimensional plot with x, y and ct axes. (Hint: Calculate the Hamiltonian associated with L and use energy conservation.)

Problem 3 [*Covariant analysis of open string endpoint motion.*]: Use the explicit form of $\mathcal{P}^\sigma_\mu$ to calculate $\mathcal{P}^\sigma_\mu \mathcal{P}^{\sigma\mu}$, and use the result of this calculation to prove that free open string endpoints move with the speed of light.