Sheet 3

Due date: 14 March 2014

Exercise 1 [*Magnetic field of an inductor*]: Consider an inductor with radius R and length L oriented along the z-direction, for which the number of windings per unit length equals n. If the electric current through the inductor is I, compute the z-component of the magnetic field for points on the symmetry axis of the inductor. What is the value of the magnetic field in the limit $L \to \infty$ with n held fixed?

Hint: $\displaystyle\int dx \, \frac{1}{\sqrt{x^2 + w^2}^{\,3}} = \frac{x}{w^2 \cdot \sqrt{x^2 + w^2}}$.

Exercise 2 [*Magnetic monopole*]: Dirac's ansatz for the vector potential of a magnetic monopole reads

$$\mathbf{A}(\mathbf{x}) = \frac{g}{4\pi} \int_L \frac{d\mathbf{l}' \wedge (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} ,$$

where the line integral is understood to be along the Dirac string L. Here we consider the case where L is the negative z-axis from $z = -\infty$ to $z = 0$.

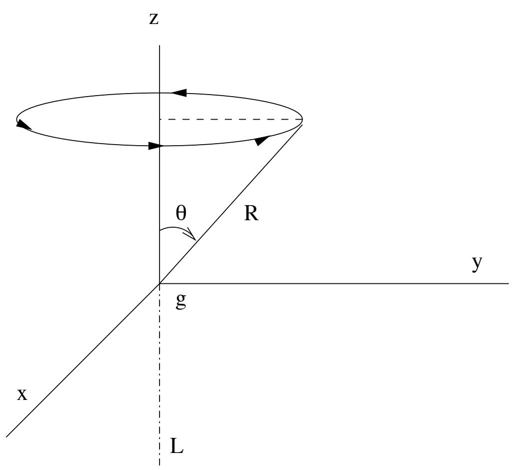
(i) Compute $\mathbf{A}$ explicitly, and show that the components of the vector potential in spherical coordinates are given by $\mathbf{A}_r = 0$, $\mathbf{A}_\theta = 0$ as well as

$$\mathbf{A}_\phi = \frac{g(1 - \cos\theta)}{r \sin\theta} = \frac{g}{r} \tan\frac{\theta}{2} .$$

(ii) Show that the magnetic field $\mathbf{B} = \nabla \wedge \mathbf{A}$ has (everywhere except for $\theta = \pi$) the same form as the Coulomb field of an electric point charge.
Hint: Use the fact that in spherical coordinates

$$\nabla \wedge (A_\phi \mathbf{e}_\phi) = \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta}(A_\phi \sin\theta)\, \mathbf{e}_r - \frac{1}{r} \frac{\partial}{\partial r}(r A_\phi) \mathbf{e}_\theta .$$

(iii) Using the result from part (ii), compute the magnetic flux through the disk of radius $R \sin\theta$ shown in the figure. Choose the orientation of the surface for both $\theta < \frac{\pi}{2}$ and $\theta > \frac{\pi}{2}$ such that the normal vector points in the positive z-direction.



(iv) Compute the line integral $\oint \mathbf{A} d\mathbf{l}$ along the circle that bounds the disk, and compare the result to the one obtained in part (iii). Why do the two results agree only for $0 < \theta < \frac{\pi}{2}$, but differ for $\frac{\pi}{2} < \theta < \pi$ by a constant term? Can you interpret this difference?