Sheet 1

Due date: 28 February 2014

Exercise 1 [*Electric potential of a hydrogen atom*]: The electric potential of a hydrogen atom is given by

$$\Phi(\mathbf{r}) = k\frac{e}{a_0}\,\mathrm{e}^{-\frac{2|\mathbf{r}|}{a_0}}\left(1+\frac{a_0}{|\mathbf{r}|}\right),$$

where e is the elementary charge and a_0 is the Bohr radius. Find the charge density distribution $\rho(\mathbf{r})$ of this potential, and verify that the hydrogen atom is electrically neutral.

[**Hint**: Use Poisson's equation as well as the identities

$$\begin{aligned}\Delta\left(\frac{1}{|\mathbf{r}|}\right) &= -4\pi\delta^{(3)}(\mathbf{r})\\ \int_0^\infty \mathrm{d}x\; x^n \mathrm{e}^{-\beta x} &= (-1)^n\frac{\partial^n}{\partial\beta^n}\left[\int_0^\infty \mathrm{d}x\;\mathrm{e}^{-\beta x}\right] = \frac{n!}{\beta^{n+1}} \qquad (\beta>0).]\end{aligned}$$

Exercise 2 [*Conducting sphere in an electric field*]: A conducting sphere with radius R and total charge Q is brought into a homogeneous electric field $\mathbf{E}^0 = E_0\mathbf{e}_3$. Compute the electric potential of this configuration.

[**Hint**: Motivate the following ansatz in spherical coordinates

$$\Phi = f_0(r) + f_1(r)\cos\theta\,,$$

and solve Laplace's equation $\Delta\Phi = 0$ with

$$\Delta\Phi(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2\Phi}{\partial\phi^2}\,.$$

To find the solution use the following boundary conditions:

(i) At infinity the electric field goes to the homogeneous electric field.

(ii) The electric potential is constant on the surface of the sphere.

The remaining parameter is determined by Gauss's law.]