Exercise 1. *Critical temperature in the Stoner model*

We consider the following dispersion relations:

(a) $\epsilon_{\boldsymbol{k}} = \epsilon_0 \pm \frac{\hbar^2 \boldsymbol{k}^2}{2m}$ (3D)

(b) $\epsilon_k = \epsilon_0 + \alpha|k|$ (1D).

Plot the critical temperature of the Stoner model for fixed interaction strength U depending on the chemical potential μ.

Exercise 2. *Stoner instability*

In the lecture, it was shown that a system described by the mean-field Hamiltonian

$$\mathcal{H}_{\mathrm{MF}} = \frac{1}{\Omega} \sum_{\boldsymbol{k},s} (\epsilon_{\boldsymbol{k}} + U n_{-s}) c^{\dagger}_{\boldsymbol{k}s} c_{\boldsymbol{k}s} - U n_{\uparrow} n_{\downarrow} \tag{1}$$

shows an instability towards a magnetically ordered state at $N(\epsilon_F)U_C = 2$
Show for the case of a parabolic dispersion and $T = 0$ that there are actually three distinct states:

- a paramagnetic state: $N(\epsilon_F)U < 2$,
- an imperfect ferromagnetic state: $3/2^{1/3} > N(\epsilon_F)U > 2$ and
- a perfect ferromagnetic state: $N(\epsilon_F)U > 3/2^{1/3}$.

Hint: Introduce a variable for the magnitude of the polarization

$$\frac{N_\uparrow}{N_e} = \frac{1}{2}(1+x) \qquad \frac{N_\downarrow}{N_e} = \frac{1}{2}(1-x) \tag{2}$$

where $N_{\uparrow(\downarrow)}$ is the total number of up-spins (down-spins) and N_e is the total number of electrons.
Minimize the total energy of the system with respect to x.
Plot the polarization of the system x as a function of $N(\epsilon_F)U$.